4.3 / Practice workbook

Rates of Change in Applied Contexts

4.3 / AP-style free response

Problem 1 ➤ */ Show the work and the reasons a grader looks for.*

People enter a museum at a rate of $E(t) = 40 + 10t$ people per hour and leave at a rate of $L(t) = 6t^2$ people per hour, where t is measured in hours after the museum opens, $0 \le t \le 5$. $N(t)$ is the number of people in the museum.

(a) Is the number of people in the museum increasing or decreasing at $t = 3$? Give a reason.

(b) Find $N''(3)$. Interpret it in context, with units.

2 The area of a circle is $A = \pi r^2$. What is the rate of change of the area with respect to the radius when $r = 4$ cm?

A. 4π cm^2/cm
B. 8π cm^2/cm
C. 16π cm^2/cm
D. 16 cm^2/cm

4.3 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

The temperature of a lake, in degrees Celsius, at a depth of x meters is $T(x)$. Selected values are in the table.

x (meters)	0	4	10	16
$T(x)$	22	19	13	10

(a) Use the table to estimate $T'(7)$.

(b) Interpret your answer to (a) in context, with units.

4 The cost of producing x items is $C(x) = 0.02x^2 + 15x + 400$ dollars. What is $C'(100)$, and what does it mean?

A. $C'(100) = 2100$: it costs 2100 dollars to make 100 items.

B. $C'(100) = 15$: each item costs 15 dollars.

C. $C'(100) = 19$: at 100 items, the cost is increasing at 19 dollars per item.

D. $C'(100) = 21$: the cost per item is 21 dollars.

4.3 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

For a gas at a fixed temperature, the pressure is $P = \frac{800}{V}$ kilopascals when the volume is V liters.

(a) Find $\frac{dP}{dV}$ when $V = 20$. Give units.

(b) Interpret your answer in context.

4.3 / Answer key

Rates of Change in Applied Contexts

1 (a) $N'(3) = E(3) - L(3) = 70 - 54 = 16 > 0$: increasing; (b) $N''(3) = -26$: at $t = 3$, the rate N' is decreasing at 26 people per hour per hour

2 (B)

3 (a) -1; (b) At a depth of 7 meters, the temperature is decreasing at about 1 degree Celsius per meter of depth

4 (C)

5 (a) -2 kilopascals per liter; (b) When the volume is 20 liters, the pressure is decreasing at 2 kilopascals per liter of volume