Lecture Notes · Unit 4 · Lecture 4 AP CED 4.3

Rates beyond motion

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: FILLING OR EMPTYING?

Rates Beyond Motion AP Calculus 4.3

Tank → in 8, out 6 gal/min

Filling or emptying?

8 − 6 = 2 gal/min → filling

FILLING AT A SLOWER RATE

Still filling?

Before

In 8, out 6 gal/min

8 − 6 = +2 gal/min

Now

In 7.5, out 6 gal/min

7.5 − 6 = +1.5 gal/min

Is the tank still filling?

2 > 1.5 > 0 → still filling, more slowly

Water in the tank → A(t)

Net rate → A′ = R − 6 gal/min

A′ > 0 → amount rises

A″ < 0 → net rate falls

A″ = R′ → gal/min per min

R′(3) = −0.5 → inflow slowing

PROBLEM 1 OF 3

In minus out

Water flows into a tank at $f(t) = 10 + 2t$ gallons per hour and out at $g(t) = t^2$ gallons per hour, $0 \le t \le 6$, t in hours. $A(t)$ is the amount of water, in gallons.

(a) Is A increasing at $t = 4$? Justify your answer.

(b) Find $g'(4)$. Interpret it.

(c) Find $A''(4)$. Interpret it.

Net rate
A′(t) = f(t) − g(t)
Units of a rate's rate
gal per hour, per hr

Solution A' = in - out; the rate of a rate

A′ = f − g → primes 3 parts, 1 rule

(a)

$A'(t) = f(t) - g(t) = 10 + 2t - t^2$

$A'(4) = 10 + 8 - 16 = 2$

A′(4) > 0 → A increasing

Try (b), (c).

Solution A' = in - out; the rate of a rate

Net rate
A′(t) = f(t) − g(t)
Units of a rate's rate
gal per hour, per hr
(a) Amount is rising
A′(4) = +2 gal/h

(b)

Plan: each prime, with units

$g'(t) = 2t \rightarrow g'(4) = 8$

At t = 4 hr, the outflow rate is increasing

at 8 gal per hour, per hr

(c)

$A'' = f' - g' = 2 - 2t$

$A''(4) = 2 - 8 = -6$

At t = 4, A′ decreasing at 6 gal per hour, per hr

$A'(4) > 0$, $A''(4) < 0 \rightarrow$ still filling

Workbook → in minus out

Careful → a rate of a rate

X Common mistake

$g'(4) = 8$ gal per hr

Those are g's units, not g′s

✓ Do this

$g \rightarrow$ gal per hr

$g' \rightarrow$ gal per hour per hour

PROBLEM 2 OF 3

A rate per centimeter

The volume of a cube with edge s cm is $V(s) = s^3$ cm³. Which is the best interpretation of $V'(4) = 48$?

(A) At $s = 4$, the volume is 48 cm³.

(B) At $s = 4$, V is increasing at 48 cm³ per cm.

(C) At $s = 4$, V grows 48 cm³ every second.

(D) At $s = 4$, the edge grows 48 cm per cm³.

Units of V′(s)

units of V per unit of s

Solution Units first → cm³ per cm

Plan: units first

Units → cm³ per cm of edge

only (B) has these units

s → edge length, not time

Workbook → units first

PROBLEM 3 OF 3

A rate from a table

Cars pass a toll booth at a rate of $C(t)$ cars per hour, t hours after 6 a.m.

(a) Use the table to estimate $C'(5)$. Show your work.

(b) Interpret your answer, with units.

t (hr)	0	2	4	6	8
C(t) (cars per hour)	120	156	180	171	150

Solution The two times around 5, then a sentence

Plan: 2 times $4 < 5 < 6$

(a)

$$C'(5) \approx \frac{C(6) - C(4)}{6 - 4}$$

$$= \frac{171 - 180}{2} = -\frac{9}{2} = -4.5$$

Solution The two times around 5, then a sentence

t (hr)	0	2	4	6	8
C(t) (cars per hour)	120	156	180	171	150

(b)

Units → cars per hour per hr; table → 'about'

At t = 5 (11 a.m.), the traffic rate is

decreasing at about 4.5 cars per hour per hr

Minus → decreasing

Workbook → two times, a sentence

WRAP-UP

Same tank, two questions

Positive net rate → amount rises

Workbook

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.