4.3 / Rates of Change in Applied Contexts

Watch the lesson ➤

Rates of change in applied contexts other than motion

Learning goal

Read rates of change in contexts other than motion: net rate in minus out, rates with respect to any variable, and table estimates with units.

Key ideas

01 A tank: in at 8 gal/min, out at 6 gal/min. Filling or emptying? In 1 minute, about +8 and −6: $A' = 8 - 6 = 2$ gal/min, $A' > 0$, **filling**. Like a bank app: the **net** change.

02 An inflow rate $R(t)$ in gal/min has R' in gal/min **per** min. $R'(3) = -0.5$: the inflow rate is decreasing at 0.5 gal/min per min (AP 2023 missed the rate of a rate).

03 Not motion, so no "velocity": say what the quantity is (bananas have no velocity).

Remember

net rate
Rate in minus rate out: $A'(t) =$ in $-$ out.

Class notes

4.3 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The rate in is not the rate of change

Code R3

✗ Water flows in at $f(4) = 18$ gal/hr, so A is increasing at 18 gal/hr.

✓ $A'(4) = f(4) - g(4) = 18 - 16 = 2$: in minus out.

4.3 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Rate in minus rate out (net rate)	A' = in − out; its sign says increasing or decreasing.	E01 · Practice 1
A rate with respect to a variable that is not time	Units of f per unit of x, whatever x is.	E02 · Practice 2, 4, 5
A rate estimated from a table, with units	The difference quotient of the two closest values around the point; then the sentence with units.	E03 · Practice 3

Example 01

Video · E01

Water flows into a tank at $f(t) = 10 + 2t$ gallons per hour and out at $g(t) = t^2$ gallons per hour, $0 \le t \le 6$, t in hours. $A(t)$ is the amount of water, in gallons.
(a) Is A increasing at $t = 4$? Why?
(b) Find $g'(4)$. Interpret it.
(c) Find $A''(4)$. Interpret it.

> $A' = f - g$, then the primes.

Example 02

Video · E02

The volume of a cube with edge s cm is $V(s) = s^3$ cm³. Which is the best interpretation of $V'(4) = 48$?

A. At $s = 4$, the volume is 48 cm³.
B. At $s = 4$, V is increasing at 48 cm³ per cm.
C. At $s = 4$, V grows 48 cm³ every second.
D. At $s = 4$, the edge grows 48 cm per cm³.

> The units first, then the meaning.

Example 03

Video · E03

Cars pass a toll booth at a rate of $C(t)$ cars per hour, t hours after 6 a.m.
(a) Use the table to estimate $C'(5)$. Show the work.
(b) Interpret your answer, with units.

t (hours)	0	2	4	6	8
$C(t)$	120	156	180	171	150

> The times around 5 are 4 and 6.

VIDEO COMPLETE Check your example answers at the end of this booklet.

4.3 / Example answers

Rates of Change in Applied Contexts

E01 (a) $A'(4) = 18 - 16 = 2 > 0$: increasing; (b) $g'(4) = 8$: the outflow rate is increasing at 8 gal/hr per hr; (c) $A''(4) = -6$: A' is decreasing at 6 gal/hr per hr

E02 (B)

E03 (a) $-\frac{9}{2}$; (b) At t = 5, the rate at which cars pass is decreasing at about 4.5 cars per hour per hour