Lecture Notes · Unit 4 · Lecture 3 AP CED 4.2b

Speed: speeding up and slowing down

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INTRO: BRAKES OR GAS?

Speeding Up or Slowing Down? AP Calculus 4.2

Last time → a question

v = −5, a = −2 → slowing down?

Gut says slower → speeding up

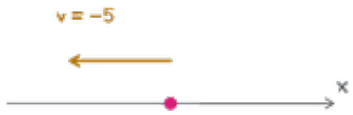
Faster or slower?

Two signs → one sentence

IDEA 1: SPEED

Speed

v = −5 → left



− → direction only

Reverse → 5, not −5

Speed

speed → |v(t)|

v = −5 → speed = |−5| = 5

IDEA 2: TWO SIGNS

Two signs

Second by second

start: v = −5, a = −2

a = v′ → v drops 2 each second

t = 0, 1, 2: v = −5, −7, −9

v	−5	−7	−9
|v|	5	7	9

speed = |v|: 5, 7, 9 → speeding up

v↓, speed ↑ ?

v < 0 → left, a < 0 → pushes left

→ faster



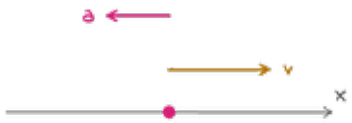
Fastest here: t = 2 → |v| = 9

Start v = 5, same a → v = 5, 3, 1

v	5	3	1
|v|	5	3	1

speed → slowing

→ slower



Same sign → same team → faster

Opposite: tug of war → slower

−5, −2 → same sign → speeding up

IDEA 3: THE AP SENTENCE

Say it the AP way

The AP way

one t: sign of v, sign of a

Show a(t) = v′(t) first

v(1) < 0 and a(1) < 0 → same sign

At $t = 1$ → speed increasing

one sign → not enough

2024 FRQ 2(b) → $a = v'$, then both signs

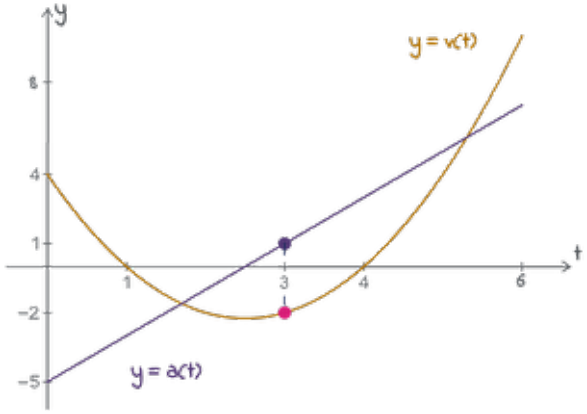
PROBLEM 1 OF 3

Speeding up or slowing down

A particle moves along a line with velocity $v(t) = t^2 - 5t + 4$ for $0 < t < 6$.

(a) At $t = 3$, is the speed increasing or decreasing? Give a reason.

(b) On what intervals is the speed increasing? Give a reason.



Solution (a) Both signs at t = 3

(a)

Plan: $a = v'$, then both signs at $t = 3$

$a = v' = 2t - 5$

$v(3) = 9 - 15 + 4 = -2 < 0$

$a(3) = 6 - 5 = 1 > 0$

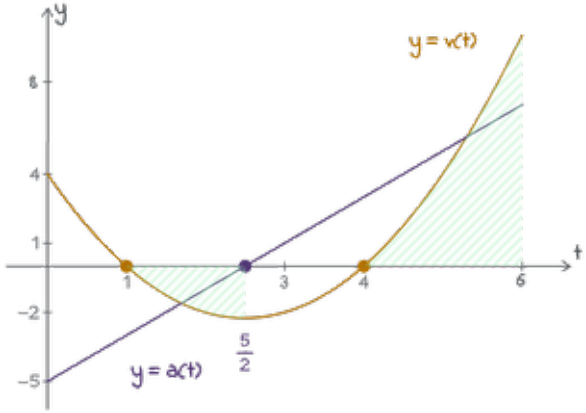
$v(3) < 0$, $a(3) > 0$ → speed decreasing

Shortcut: $a(3) > 0$ → faster? No.

$v(3) < 0$ → left, push right → slowing → two signs

Your turn: part (b)

Solution (b) Match the signs



(b)

Plan: signs of v, signs of a → match

$v = (t - 1)(t - 4)$

$v < 0$ on $(1, 4)$; $v > 0$ on $(0, 1)$, $(4, 6)$

$a = 2t - 5 = 0$ at $\frac{5}{2}$ → $a < 0$, then $a > 0$

Same: $\left(1, \frac{5}{2}\right)$ both −, $(4, 6)$ both +

$(0, 1)$, $\left(\frac{5}{2}, 4\right)$ → opposite

on $\left(1, \frac{5}{2}\right)$, $(4, 6)$ → speed increasing

Workbook → both signs

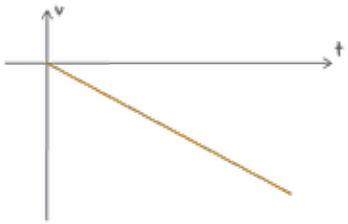
IDEA 4: ON A GRAPH OF V

On a graph of v

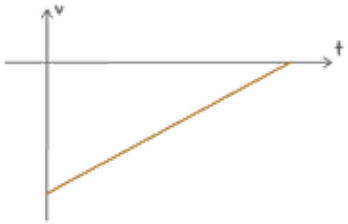
Speed on a graph

$|v|$ = distance from the t-axis

Away → speeding up



Toward → slowing down



PROBLEM 2 OF 3

Speed from a velocity graph

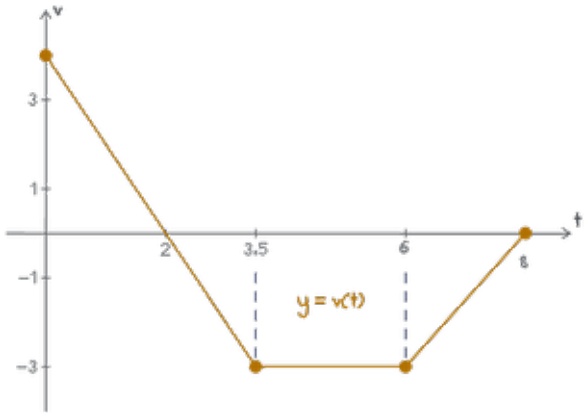
The graph of the velocity $v(t)$ of a particle is shown for $0 \le t \le 8$. On which interval is the speed of the particle increasing?

(A) $0 < t < 2$

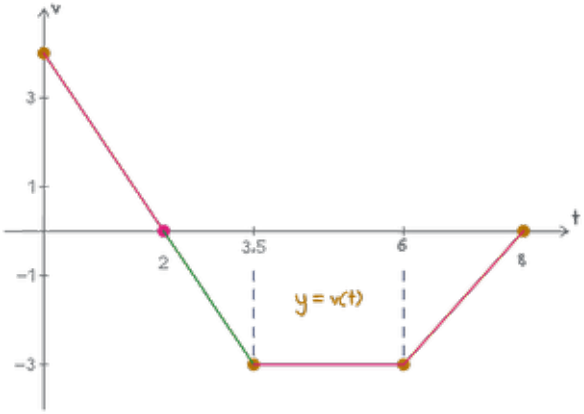
(B) $2 < t < 3.5$

(C) $3.5 < t < 6$

(D) $6 < t < 8$



Solution Away from the t-axis = speeding up



Plan: away → faster

(A) toward → slower

(B) away → faster

(C) flat → speed 3

(D) toward → slower

So (B)

Workbook → same question

PROBLEM 3 OF 3

Direction and speed at t = 1

A particle moves along a line with velocity $v(t) = e^{-t}(t-2)$. At $t = 1$, the particle is

(A) moving left and speeding up

(B) moving left and slowing down

(C) moving right and speeding up

(D) moving right and slowing down

Product rule

$(fg)' = f'g + fg'$

Chain rule

$\frac{d}{dt}e^{-t} = -e^{-t}$

Solution Sign of v, then a by the product rule

Plan: sign of $v(1)$, then sign of $a(1)$

$$v(1) = e^{-1}(1-2) = \frac{1}{e}\cdot(-1) = -\frac{1}{e} < 0 \rightarrow \text{left}$$

$$a(t) = -e^{-t}(t-2) + e^{-t}\cdot 1$$

$$= e^{-t}(-(t-2)+1) = e^{-t}(-t+2+1)$$

$$= e^{-t}(3-t)$$

$$a(1) = e^{-1}\cdot 2 = \frac{2}{e} > 0$$

$v < 0$, $a > 0$ → slowing down

Left, slowing → (B)

Workbook → two signs

WRAP-UP

Faster or slower?

$v = -5$, $a = -2$ → same sign → speeding up

Same sign → faster. Opposite → slower.

Show $a = v'$, both signs at one t, the word speed

Workbook → comments → next: 4.3

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.