Lecture Notes · Unit 4 · Lecture 2 AP CED 4.2a

Straight-line motion

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: STOP, OR TURN?

Does v = 0 Mean It Turns Around? AP Calculus 4.2

on a line: v(2) = 0 → it stops



So it turns around at t = 2?

Stop, or turn?

VELOCITY AND DIRECTION

Position, velocity, direction

on a line → position x(t)

Velocity

v(t) = x′(t) (rate of change of x)

x in ft, t in sec → v in ft/sec

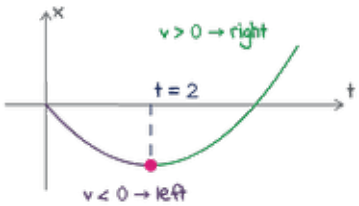
Sign of v → direction

v > 0 → moving right

v < 0 → moving left

v = 0 → stopped

$x = t^2 - 4t \rightarrow v = 2t - 4$



x(0) = 0, x(2) = −4, x(4) = 0



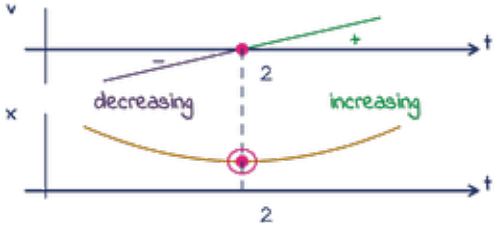
Vertical line → up is +, same rule

STOP, OR TURN AROUND?

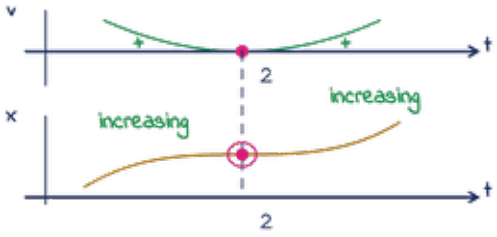
Stop, or turn around?

v(2) = 0

v = t − 2 → turns at 2



$v = (t-2)^2$ → same way, no turn



Turns where v changes sign

v changes sign at t = 2.

ACCELERATION

When v changes

v(t) = 2t + 1

t	1	2	3
v(t)	3	5	7
	+2	+2	

Each second → v up 2

That 2 → acceleration

$a(t) = v'(t) = x''(t)$ → here 2

$a = -2$ → v goes down 2 each sec

Units → ft/sec per sec

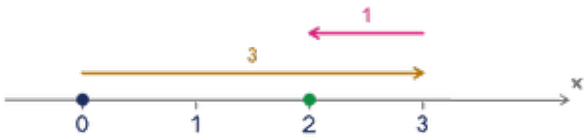
Average a → avg. rate of change of v

$$\frac{v(3)-v(1)}{3-1} = \frac{7-3}{2} = 2$$

Setup: $a(3) = v'(3) = \ldots$

DISTANCE TRAVELED

Distance traveled
add the legs



Walk 3 ft right, then 1 ft left.

Displacement → $2 - 0 = 3 - 1 = 2$ ft

$3 + 1 = 4$ ft → distance

Split at each turn, then add.

PROBLEM 1 OF 3

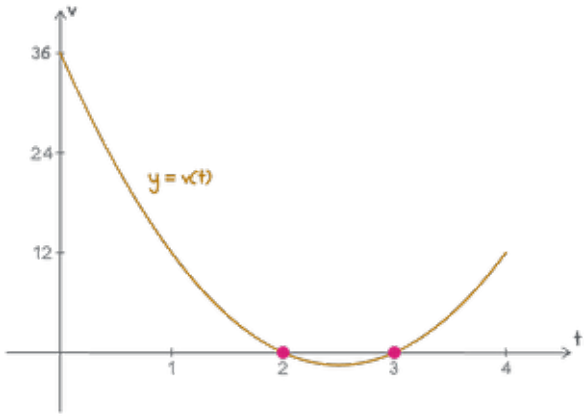
Direction, acceleration, distance

A particle moves along the x-axis with position $x(t) = 2t^3 - 15t^2 + 36t$ for $0 \le t \le 4$.

(a) When is the particle moving left? Justify your answer.

(b) Find $a(3)$.

(c) Find the total distance traveled.



Solution (a) The signs of v

Plan: signs of v, then $a = v'$, then the legs.

(a)

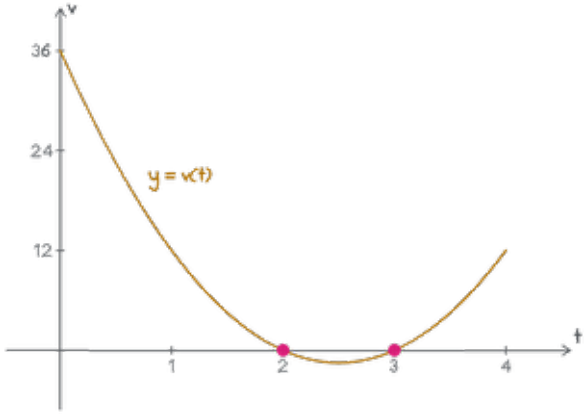
$$v = x' = 6t^2 - 30t + 36 = 6(t^2 - 5t + 6)$$

$$= 6(t-2)(t-3)$$

$v = 0$ at 2, 3 → between: $v < 0$

$2 < t < 3$ → moving left, since $v < 0$

Solution (b) a = v'



(b)

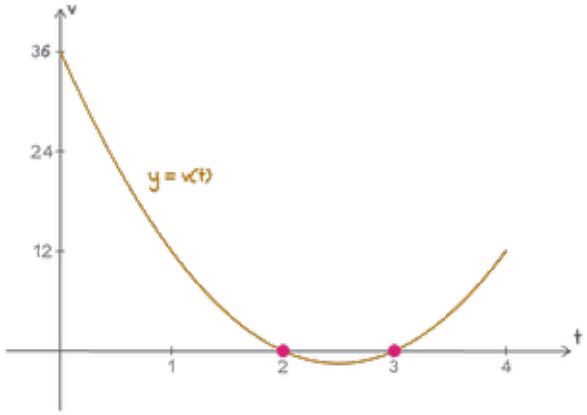
$a(t) = v'(t) = 12t - 30$

$a(3) = v'(3) = 12(3) - 30 = 6$

Not $a(t) = 6$: $a(t) = 12t - 30$, $a(3) = 6$

Your turn: part (c), the legs.

Solution (c) Split at the turns, add the legs



(c)

Turns at 2, 3 → 3 legs

$x(0) = 0$, $x(2) = 28$

$x(3) = 27$

$x(4) = 32$

$|28 - 0| + |27 - 28| + |32 - 27| = 28 + 1 + 5 = 34$

Distance 34, not 32

Practice set → v, a, legs

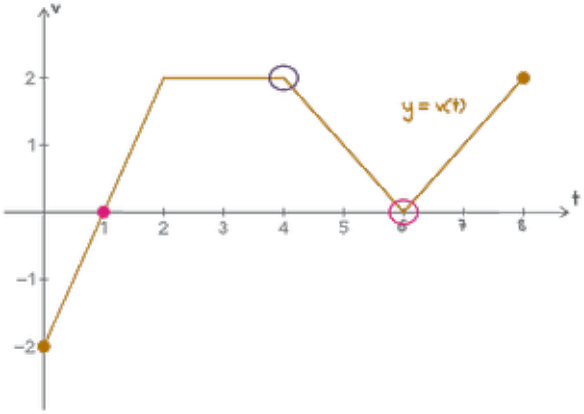
PROBLEM 2 OF 3

Reading a velocity graph

The graph of the velocity $v(t)$ of a particle, $0 \le t \le 8$, is made of segments.

(a) When in $0 < t < 8$ does the particle change direction? Justify your answer.

(b) Find $a(3)$ and $a(5)$.



Solution (a) Signs of v for direction

Plan: v, not x. Sign → direction, slope → a.

(a)

$v < 0$ on $(0, 1)$, $v > 0$ on $(1, 8)$ except

$v(6) = 0$

$(4, 6)$ → down, but $v > 0$ → still right

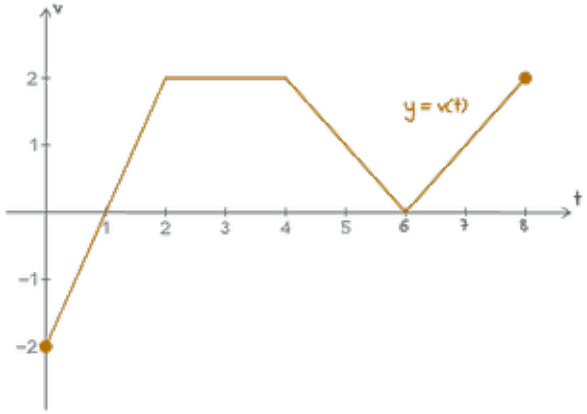
$t = 1$ → sign change

$t = 6$ → $v = 0$, no sign change

only $t = 1$

Your turn: part (b).

Solution (b) Slopes of v for acceleration



(b)

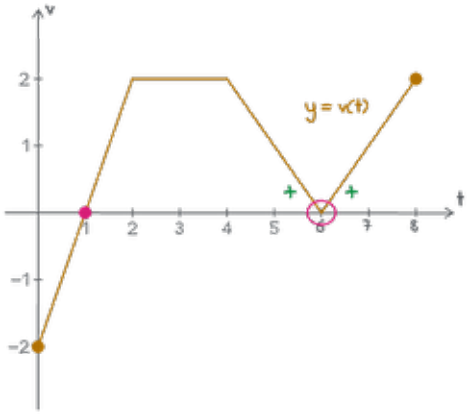
Plan: a = slope of v

$a(3) = v'(3) = 0 \rightarrow$ flat on $2 < t < 4$

$$a(5) = v'(5) = \frac{0-2}{6-4} = -1$$

Practice set → signs, slopes

v = 0 is not the reason



X Common mistake

$v(1) = v(6) = 0 \rightarrow$ turns

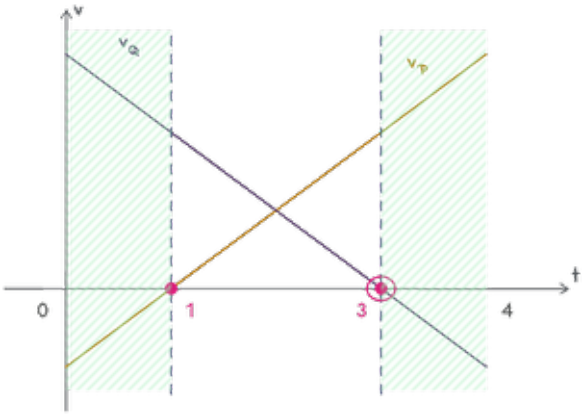
✓ Do this

Sign change → turn

PROBLEM 3 OF 3

Two particles

Two particles, P and Q, move along the x-axis for $0 \le t \le 4$. Their positions are $x_P(t) = t^2 - 2t$ and $x_Q(t) = 6t - t^2$. On what intervals in $0 < t < 4$ are the particles moving in opposite directions? Justify your answer.



Solution

Both velocities, the sign of each on every interval, then compare

Plan: v_P, v_Q → signs on $0 < t < 4$ → compare

$v_P = 2t - 2$, $v_Q = 6 - 2t$

$v_P = 0$ at $1 \rightarrow -$ then $+$

$v_Q(3) = 0 \rightarrow +$ then $-$

$(0, 1) \rightarrow v_P < 0, v_Q > 0 \rightarrow$ opposite

$(1, 3) \rightarrow$ both $> 0 \rightarrow$ same

$(3, 4) \rightarrow v_P > 0, v_Q < 0 \rightarrow$ opposite

Opposite on $0 < t < 1$ and $3 < t < 4 \rightarrow$ opposite signs.

Practice set → every interval

WRAP-UP

Stop, or turn?

Turns only if v changes sign

Two particles → every interval

$a = v'$ → show $a(3) = v'(3)$

Practice set → sign by sign

Next → speed up, slow down

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.