4.2a / Straight-Line Motion: Velocity, Acceleration, Direction

Watch the lesson ➤

Connecting position, velocity, and acceleration · Part 1

Learning goal

Use position, velocity and acceleration on a line, tell the direction from the sign of v, and find the total distance from positions.

Key ideas

01 Position $x(t)$ on a line; velocity $v = x'$. With x in ft and t in sec, v is in ft/sec. $v > 0$: right; $v < 0$: left; $v = 0$: at rest, for that instant. E.g. $x = t^2 - 4t$, $v = 2t - 4$: left for $t < 2$, right for $t > 2$.

02 Stop, or **turn**? $v = t - 2$ goes − then +: it turns. $v = (t - 2)^2$ goes +, 0, +: no turn (a red light, not a U-turn). It turns where v **changes** sign; write "v changes sign at $t = 2$."

03 $v = 3, 5, 7$ each second: v goes up 2 each second. That 2 is the **acceleration**: $a(t) = v'(t) = x''(t)$, in ft/sec per sec. Write the setup, $a(3) = v'(3) = \ldots$: the number alone earned 0 on AP 2023.

04 Walk 3 ft right, then 1 ft left: displacement $3 - 1 = 2$ ft, distance $3 + 1 = 4$ ft (like a step counter, no direction). Split at each **turn**, then add.

Remember

velocity
$v(t) = x'(t)$: its sign is the direction.

acceleration
$a(t) = v'(t) = x''(t)$: how fast the velocity changes.

total distance
The sum of the lengths of the legs between turns, never negative.

Class notes

4.2a / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 $v = 0$ is not always a turn

Code M1

✗ $v(6) = 0$, so the particle changes direction at $t = 6$.

✓ It turns only where v changes sign. Check v on both sides of $t = 6$.

02 Distance is not displacement

Code M2

✗ Distance on $[0, 4]$ is $x(4) - x(0)$.

✓ Split at each turn and add the absolute changes.

4.2a / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Direction from the sign of v (and where it changes sign)	$v = x'$; moving right when $v > 0$, left when $v < 0$; it turns where v changes sign.	E01, E02 · Practice 1, 3, 6
Acceleration with its setup	Write $a(t) = v'(t)$, then the value, with units per unit per unit.	E03 · Practice 4, 5
Total distance from positions (split at each turn)	Find the turns; add $\lvert x(\text{end}) - x(\text{start})\rvert$ for each leg.	Practice 2

Example 01

Video · E01

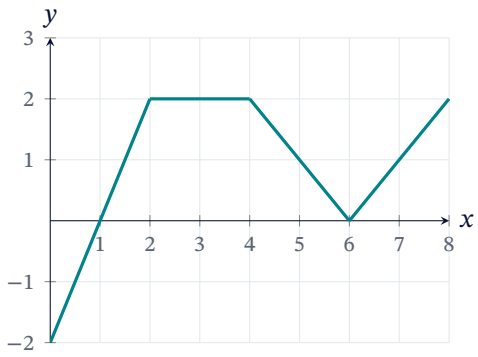
A particle moves along the x-axis with position $x(t) = 2t^3 - 15t^2 + 36t$ for $0 \le t \le 4$.
(a) When is the particle moving left? Justify.
(b) Find $a(3)$.
(c) Find the total distance traveled.

> Signs of v; then $a = v'$; then the legs between turns.

Example 02

Video · E02

The graph of the velocity $v(t)$ of a particle, $0 \le t \le 8$, is made of segments.
(a) When in $0 < t < 8$ does the particle change direction? Justify.
(b) Find $a(3)$ and $a(5)$.



Signs of v for direction; slopes for a.

Example 03

Video · E03

Two particles, P and Q, move along the x-axis for $0 \le t \le 4$. Their positions are $x_P(t) = t^2 - 2t$ and $x_Q(t) = 6t - t^2$. On what intervals in $0 < t < 4$ are the particles moving in opposite directions? Justify.

Plan: v_P, v_Q → signs on all of $0 < t < 4$ → compare

VIDEO COMPLETE Check your example answers at the end of this booklet.

4.2a / Example answers

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E01 (a) $2 < t < 3$, since $v(t) = 6(t-2)(t-3) < 0$ there; (b) 6; (c) 34

E02 (a) only $t = 1$: v changes sign there; at $t = 6$, $v = 0$ but $v > 0$ on both sides; (b) $a(3) = 0$ (flat), $a(5) = -1$

E03 0<t<1 and 3<t<4