Lecture Notes · Unit 4 · Lecture 1 AP CED 4.1

Derivative in context

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INTRO: A COCOA POPSICLE?

Derivative in Context AP Calculus 4.1

Rules → now: what does f′ mean?

Hot cocoa: T °F, t minutes

T′(5) = −3 → −3 °F? popsicle?

No. But one extra minus → the opposite

Today: one sentence, with units

UNITS, AND A RATE IS NOT AN AMOUNT

Hot cocoa: amount or rate?

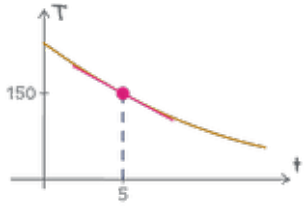
$f' \to \frac{\text{units of } f}{\text{units of } x}$

T → °F, t → min → T′ → degrees per minute

T(5) = 150 → the cocoa is 150 °F

T′(5) = −3 → cooling at 3 degrees per minute

Height → T(5). Slope → T′(5).



not −3 °F → cooling

ONE SENTENCE, FOUR PIECES

T′(5) = −3 → one sentence

t = 5 min → when

what → temperature

which way → decreasing

How fast → 3 °F per min

Four pieces

when, what, which way, how fast + units

−3 → decreasing

Decreasing at 3, or rate −3

2 minus signs → warming?

PROBLEM 1 OF 3

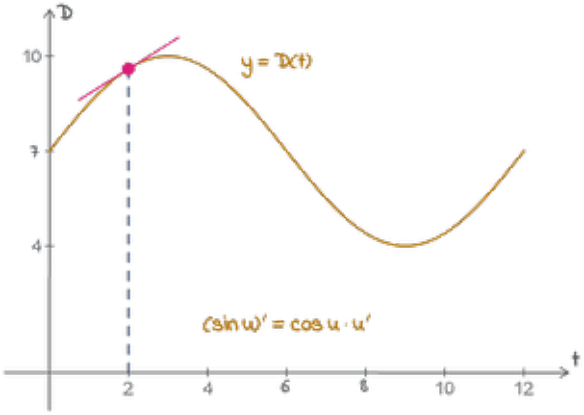
Interpret a rate with units

The depth of water in a harbor is $D(t) = 7 + 3\sin\left(\frac{\pi t}{6}\right)$ feet, t hours after midnight, $0 \le t \le 12$.

(a) What are the units of $D'(t)$?

(b) Find $D'(2)$. Interpret it in context.

(c) Find $D'(8)$. Is the water rising or falling at $t = 8$?



Solution Units, the chain rule, a sentence in four pieces

Plan: units → D′(t) → a sentence

(a)

D feet, t hours → D′ in feet per hour

(b)

7 → 0; 3sin → chain rule

$$D'(t) = 3\cos\frac{\pi t}{6}\cdot\frac{\pi}{6}$$

$$= \frac{\pi}{2}\cos\frac{\pi t}{6}$$

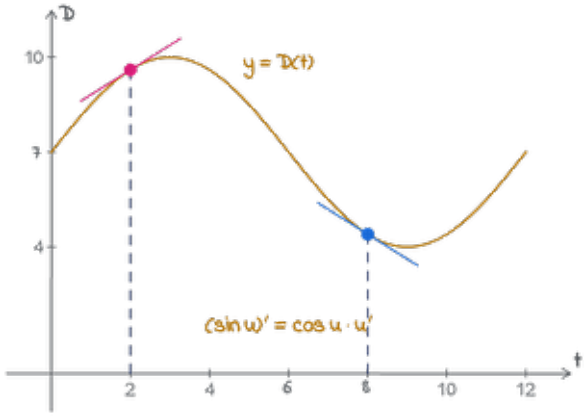
$$D'(2) = \frac{\pi}{2}\cos\frac{\pi}{3} = \frac{\pi}{2}\cdot\frac{1}{2} = \frac{\pi}{4}$$

Your turn: the sentence

At t = 2 hr, water depth

is increasing at $\frac{\pi}{4}$ ft/hr.

Solution Units, the chain rule, a sentence in four pieces



(c)

$$D'(8) = \frac{\pi}{2}\cos\frac{4\pi}{3} = \frac{\pi}{2}\left(-\frac{1}{2}\right) = -\frac{\pi}{4}$$

D′(8) < 0 → falling

Falling at $\frac{\pi}{4}$ ft/hr → no minus

A student: Decreasing at $-\frac{\pi}{4}$ ft/hr

Two minuses → rising?

Decreasing at $\frac{\pi}{4}$ ft/hr, or rate $-\frac{\pi}{4}$ ft/hr

Never both

Practice set → four pieces

NOT ONLY TIME, AND A RATE OF A RATE

Two twists

Not only time

C(x) dollars, x shirts

4.5 dollars/shirt at 100 shirts

At 100 shirts: cost increasing

at 4.50 dollars per shirt

t	5	6	7
T′(t)	−3	−2.8	−2.6

+0.2

Cocoa again

T′(t) minute by minute

Each minute → T′ up 0.2

So T″(5) ≈ 0.2 → the rate of T′

$\frac{\text{deg per min}}{\text{min}}$ → per min, per min

T″ > 0 → T′ is increasing

−3 → 0

→ still cooling, more slowly

f(t) gal/min, t min → f′ in gal/min per min

PROBLEM 2 OF 3

The best interpretation

A car gets $E(v)$ miles per gallon when it goes v miles per hour. Which of the following is the best interpretation of $E'(50) = -0.4$? At $v = 50$,

(A) the car gets 0.4 mpg.

(B) E is decreasing at 0.4 mpg per mph.

(C) E drops 0.4 over the next 50 mph.

(D) the car uses 0.4 gallons per mile.

Units of E'(v)

$\frac{\text{units of E}}{\text{units of v}}$

Four pieces

when, what, which way, how fast

Solution Four pieces, then check each choice

Plan: units → 4 pieces → choices

E → mpg, v → mph

E′ → miles per gallon, per mph

$\frac{\text{units of E}}{\text{units of v}}$

At v = 50, E decreasing at 0.4 mpg per mph

(A) reads E′ as E → 0.4 is a rate

(C) next 50 mph → one speed, v = 50

(D) gallons per mile → wrong units

So (B).

Practice set → units → four pieces

PROBLEM 3 OF 3

The rate of a rate

$P(t)$ is the population of a town t years after 2010. $P'(10) = 300$ and $P''(10) = -20$. Which must be true in 2020?

(A) $P(10) = 300$

(B) P is growing, but more slowly

(C) P is shrinking

(D) exactly 280 more people by 2021

Rates

P′(t) → rate of change of P

P″(t) → rate of change of P′

Solution P' is the rate of P; P" is the rate of P'

Plan: P′ → P, P″ → P′

t = 10 → 2020

P′(10) = 300 > 0 → growing 300 people/year.

P″(10) = −20 < 0 → P′ decreasing at 20 per yr, per yr

Still growing more slowly

Your turn: the other three

Solution P' is the rate of P; P" is the rate of P'

Rates

P′(t) → rate of change of P

P″(t) → rate of change of P′

Plan: test A, C, D.

(A) P(10) = 300 → 300 is P′, not P

(C) shrinking → P′ > 0, still growing

(D) 280 = 300 − 20 → not from one instant

So (B) → at t = 10: growing rate decreasing.

Practice set: one more → one level at a time

WRAP-UP

The cocoa

No → T'(5) is a rate: cooling

When, what, which way, how fast + units; one minus

A rate of a rate → one more per

Practice set → Next: 4.2, straight-line motion

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.