4.1 / The Meaning of the Derivative in Context

Watch the lesson ➤

Interpreting the meaning of the derivative in context

Learning goal

Read a derivative in context with units, write the sentence that earns the point (when, what, which way, how fast), and read a rate of a rate.

Key ideas

01 Cocoa: T in °F, t in minutes, $T'(5) = -3$. Is the cocoa at -3°F? T is degrees and t is minutes, so T' is degrees per **minute**. $T(5) = 150$: the cocoa is 150°F. $T'(5) = -3$: cooling at 3 degrees per minute. The height is $T(5)$; the slope is $T'(5)$.

02 The sentence: when ($t = 5$ min), what (the temperature), which way (**decreasing**), how fast (3 degrees per minute). "Decreasing at 3" or "changing at -3"; "decreasing at -3" has two minuses and earns 0.

03 Not only time: $C'(100) = 4.5$ in dollars per T-shirt: at 100 shirts, the cost is increasing at 4.50 dollars per shirt.

04 The rate of a rate: if $T' = -3, -2.8, -2.6$ at $t = 5, 6, 7$, it goes up 0.2 each minute, so $T''(5) \approx 0.2$, in degrees per minute **per** minute. $T'' > 0$: T' is increasing, from -3 toward 0: cooling more slowly.

Remember

rate of change

$f'(a)$: how fast f changes at $x = a$, in units of f per unit of x.

rate of a rate

$f''(a)$: how fast the rate f' changes, in units of f per unit of x per unit of x.

Class notes

4.1 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Decreasing at a positive number

Code R1

✗ At $t = 5$ the temperature is decreasing at a rate of -3 degrees per minute.

✓ Decreasing at a rate of 3 degrees per minute (or changing at -3). Two minuses earn 0.

02 A rate is not the amount

Code R2

✗ $T'(5) = -3$, so the cocoa is -3°F.

✓ $T(5)$ is the temperature; $T'(5)$ is how fast it changes: cooling at 3 degrees per minute.

4.1 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Units of a derivative	Units of f per unit of x.	Practice 3
A rate in a sentence (when, what, which way, how fast, units)	At $t = a$ (when), the quantity (what) is increasing/decreasing (which way) at a rate of $\lvert f'(a)\rvert$ (how fast) units.	E01, E02 · Practice 1, 2, 5
The rate of a rate	$f''(a)$ is about f': the rate is increasing or decreasing, in units per unit per unit.	E03 · Practice 4

Example 01

Video · E01

The depth of water in a harbor is $D(t) = 7 + 3\sin\left(\frac{\pi t}{6}\right)$ feet, t hours after midnight, $0 \le t \le 12$.
(a) What are the units of $D'(t)$?
(b) Find $D'(2)$. Interpret it in context.
(c) Find $D'(8)$. Is the water rising or falling at $t = 8$?

> Units first. Then a sentence with when, what, which way, how fast.

Example 02

Video · E02

A car gets $E(v)$ miles per gallon when it goes v miles per hour. Which is the best interpretation of $E'(50) = -0.4$? At $v = 50$,

A. the car gets 0.4 mpg.
B. E is decreasing at 0.4 mpg per mph.
C. E drops 0.4 over the next 50 mph.
D. the car uses 0.4 gallons per mile.

> The units of E' first.

Example 03

Video · E03

$P(t)$ is the population of a town t years after 2010. $P'(10) = 300$ and $P''(10) = -20$. Which must be true in 2020?

A. $P(10) = 300$
B. P is growing, but more slowly
C. P is shrinking
D. exactly 280 more people by 2021

> P' is about P. P'' is about P'.

VIDEO COMPLETE Check your example answers at the end of this booklet.

4.1 / Example answers

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E01 (a) feet per hour; (b) $D'(2) = \frac{\pi}{4}$: at $t = 2$ hours the depth is increasing at $\frac{\pi}{4}$ feet per hour; (c) $D'(8) = -\frac{\pi}{4} < 0$: falling, at $\frac{\pi}{4}$ feet per hour

E02 (B)

E03 (B)