3.6 / Practice workbook

Calculating Higher-Order Derivatives

Practice

Problems 1–8 / Show your reasoning

1 Find each second derivative. (a) If $f(x) = \sqrt{x}$, find $f''(4)$. (b) If $y = \tan x$, find $\dfrac{d^2y}{dx^2}$ at $x = \dfrac{\pi}{4}$.

2 Find each higher-order derivative.
(a) $\dfrac{d^{23}y}{dx^{23}}$ for $y = \cos x$
(b) y''' for $y = e^{-3x}$

3 If $f(x) = \cos(\pi x)$, then $f''\left(\dfrac{1}{3}\right) =$

A. $-\dfrac{\pi}{2}$
B. $-\dfrac{\pi^2}{2}$
C. $\dfrac{\pi^2}{2}$
D. $-\dfrac{\sqrt{3}\,\pi^2}{2}$

4 If $y = x \ln x$, what is the value of $\dfrac{d^2y}{dx^2}$ at $x = 2$?

A. $\ln 2 + 1$
B. $\dfrac{1}{2}$
C. 1
D. 0

5 If $y^2 = x$ and $y \neq 0$, then $\dfrac{d^2y}{dx^2} =$

A. $-\dfrac{1}{2y^2}$
B. $\dfrac{1}{2y}$
C. $-\dfrac{1}{4y^3}$
D. 0

3.6 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

Consider the curve given by $y^2 - x^2 = 3$.

(a) Show that $\frac{dy}{dx} = \frac{x}{y}$.

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Then use the equation of the curve to write it in terms of y only.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 2)$.

3.6 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

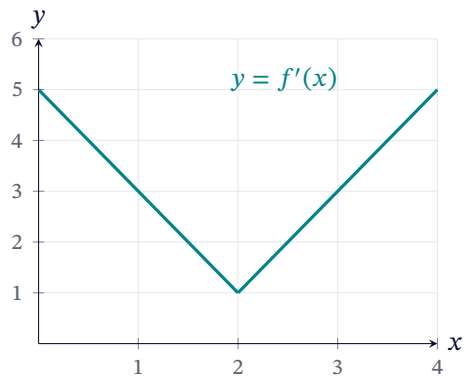
The table gives values of a twice-differentiable function f and its first two derivatives.

x	0	2
$f(x)$	0	5
$f'(x)$	2	3
$f''(x)$	-3	-1

(a) Let $h(x) = x\,f(x)$. Find $h''(2)$.

(b) Let $k(x) = e^{f(x)}$. Find $k''(0)$.

8 The graph of f', made of line segments, is shown, and $f(1) = 2$. Let $h(x) = [f(x)]^2$. Then $h''(1) =$



A. 10
B. 14
C. 18
D. 26

3.6 / Answer key

Calculating Higher-Order Derivatives

1 (a) $-\frac{1}{32}$; (b) 4

2 (a) $\sin x$; (b) $-27e^{-3x}$

3 (B)

4 (B)

5 (C)

6 (a) x/y; (b) $\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{y^3} = \frac{3}{y^3}$; (c) $\frac{3}{8}$

7 (a) $h''(2) = 2f'(2) + 2f''(2) = 6 - 2 = 4$; (b) $k''(0) = e^{f(0)}\left[f'(0)\right]^2 + e^{f(0)}f''(0) = 4 - 3 = 1$

8 (A)