Lecture Notes · Unit 3 · Lecture 8 AP CED 3.6

Higher-Order Derivatives and Implicit Differentiation | AP Calculus AB 3.6

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Today → differentiate f'. what comes out?

$f(x) = x^4 - 3x^2$ curve → $\frac{dy}{dx}$

$f(x) = x^4 - 3x^2$

$f'(x) = 4x^3 - 6x$

f' differentiable

$12x^2 - 6 = f''(x)$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$

top d d y, bottom dx dx → $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} \neq \left(\frac{dy}{dx}\right)^2$

$f''', f^{(4)}, \ldots, \frac{d^ny}{dx^n}$

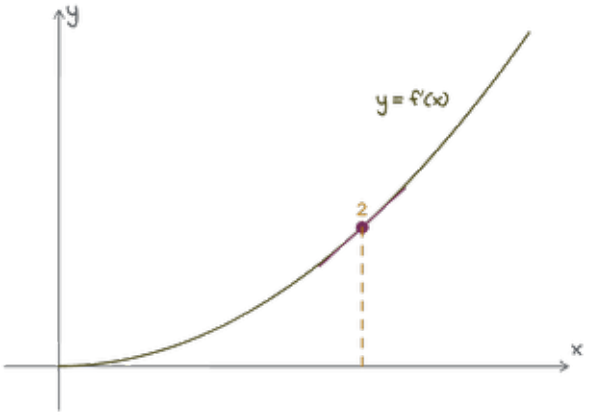
$x^4 \to 4x^3 \to 12x^2 \to 24x \to 24 \to 0$

$f(x) = x^4 \to f^{(5)} = 0$

sin → cos → −sin → −cos → sin

$g(x) = \sin x \to g^{(8)} = g^{(4)} = \sin x$

$2e^{2x}, 4e^{2x}, 8e^{2x} \to 2^n e^{2x}$



$f = e^{x^2} \to f' = 2xe^{x^2}$

f' is a product

$f''(x) = 2e^{x^2} + 2x \cdot 2xe^{x^2}$

$x = 2 \to$ slope $2 = f''(2)$

Height → f' slope → f''

PROBLEM 1 OF 3

If $f(x) = xe^{2x}$, then $f''(0) =$

(A) 1 (B) 2 (C) 4 (D) 12

$(uv)' = u'v + uv'$

$(e^{2x})' = 2e^{2x}$

Solution

$f' \to f'' \to x = 0$

$f'(x) = e^{2x} + 2xe^{2x}$

$= e^{2x}(1 + 2x)$

$f''(x) = 2e^{2x}(1 + 2x) + 2e^{2x}$

$f''(0) = 2 + 2 = 4$

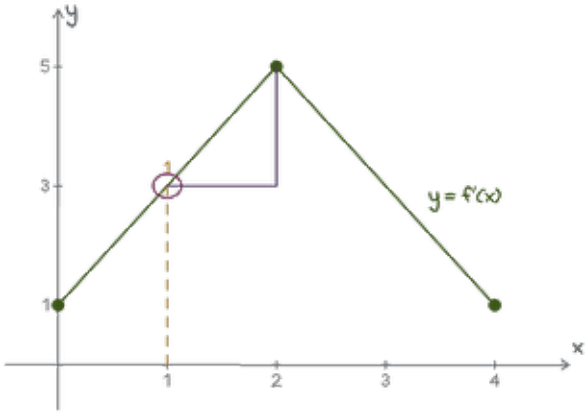
Workbook

PROBLEM 2 OF 3

The graph of f', made of line segments, is shown. Let $g(x)=f(x^2)$. Then $g''(1)$ =

(A) 4 (B) 6

(C) 8 (D) 14



Solution

g' chain $\cdot\ x \cdot x = 1$

$g'(x) = f'(x^2) \cdot 2x$

$(f'(x^2))' = f''(x^2) \cdot 2x$

$f''(x^2)2x \cdot 2x + f'(x^2)2$

$g''(1) = 4f''(1) + 2f'(1)$

$f'(1) = 3$

$f''(1) = 2$

$g''(1) = 8 + 6 = 14$

Workbook

$x^2 - y^2 = 9, \quad \frac{dy}{dx} = \frac{x}{y}$

$y'' = \frac{y \cdot 1 - xy'}{y^2}$

$= \frac{y - x \cdot \frac{x}{y}}{y^2}$

$= \frac{y^2 - x^2}{y^3}$

$= \frac{-9}{y^3}$

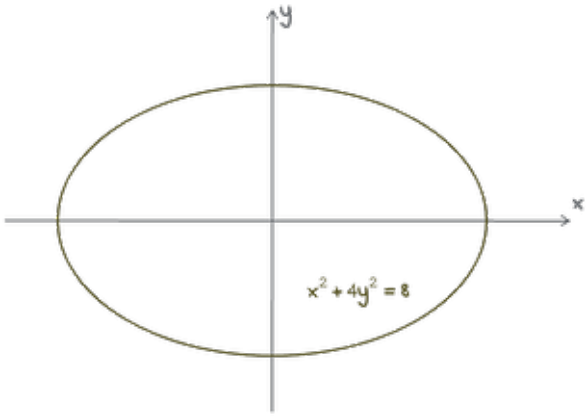
$D_x \rightarrow y' \rightarrow$ use the curve

PROBLEM 3 OF 3

Consider the curve $x^2 + 4y^2 = 8$.

(a) Show that $\frac{dy}{dx} = -\frac{x}{4y}$.

(b) Find $\frac{d^2y}{dx^2}$ at $(2, 1)$.



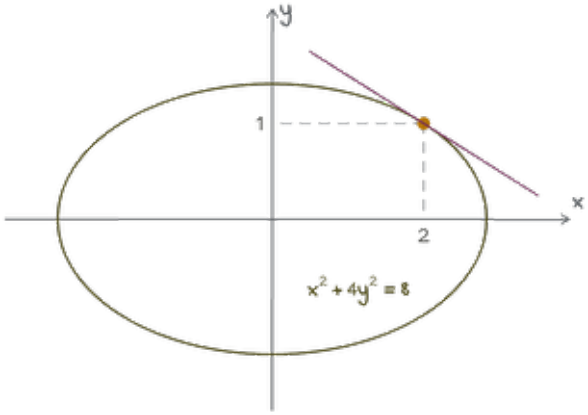
Solution

(a)

$2x + 8y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x}{8y} = -\frac{x}{4y}$

Solution



(b)

$y'(2, 1) \rightarrow y'' \rightarrow$ values

$$-\frac{2}{4} = -\frac{1}{2}$$

$$y'' = -\frac{4y \cdot 1 - x\left(4\frac{dy}{dx}\right)}{(4y)^2}$$

$$-\frac{4 - 2 \cdot 4\left(-\frac{1}{2}\right)}{16}$$

$$= -\frac{8}{16} = -\frac{1}{2}$$

Workbook

y is not a constant

X Common mistake

$$-\frac{x}{4y} \rightarrow -\frac{1}{4y}$$

$$-\frac{1}{4}$$

✓ Do this

y is not a constant $\rightarrow 4y' \rightarrow -\frac{1}{2}$

2015 AP $\rightarrow \frac{d^2y}{dx^2} \rightarrow$ 4 points

f'' → Unit 4 motion, Unit 5 shape

Next → Unit 4

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.