3.6 / Calculating Higher-Order Derivatives

Watch the lesson ➤

Higher-order derivatives

Learning goal

Find second and higher derivatives from formulas, curves given implicitly, and graphs of f'.

Key ideas

01 Differentiate a derivative: $f(x) = x^4 - 3x^2$, $f'(x) = 4x^3 - 6x$, $\frac{d}{dx}f'(x) = 12x^2 - 6$: the **second** derivative $f''(x)$, also $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$, read "d **two** y, d x squared". Next: $f'''(x)$, $f^{(4)}(x)$, $\frac{d^ny}{dx^n}$.

02 Patterns: for x^4 the power goes **down** by 1 each step, so $f^{(5)}(x) = 0$. $\sin x$ comes back after **four**: $f^{(8)} = f^{(4)} = \sin x$. $e^{2x} \to 2e^{2x} \to 4e^{2x} \to 8e^{2x}$, so $\frac{d^ny}{dx^n} = 2^ne^{2x}$.

03 $f(x) = e^{x^2}$: $f'(x) = 2x\,e^{x^2}$ is a **product**, so $f''(x) = 2e^{x^2} + 4x^2e^{x^2}$.

04 From a graph of f': the slope of f' at 2 is $f''(2)$.

05 A curve: $x^2 + y^2 = 25$, $\frac{dy}{dx} = -\frac{x}{y}$, then $\frac{d^2y}{dx^2} = -\frac{y - x\frac{dy}{dx}}{y^2} = -\frac{y^2 + x^2}{y^3} = -\frac{25}{y^3}$. Put in $\frac{dy}{dx}$, then use the **curve**.

Remember

second derivative
The derivative of the derivative: $f''(x)$ or $\frac{d^2y}{dx^2}$.

Class notes

3.6 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 y is not a constant

Code I1

✗ $\frac{d}{dx}\left(-\frac{x}{4y}\right) = -\frac{1}{4y}.$

✓ y depends on x: the quotient rule, with $\frac{dy}{dx}$ put in.

3.6 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
A second derivative from a formula, at a point	$f'' = \frac{d}{dx} f'$: find f', simplify it, then differentiate f' with the same rules. Put the number in last.	E01 · Practice 1, 3, 4
Patterns in higher derivatives	Write the first few derivatives until the pattern shows: sin and cos come back after four, e^{kx} gains a factor k each time.	Practice 2
Second derivative of a curve given implicitly	Differentiate $\frac{dy}{dx}$ (quotient rule, y is still a function of x), put $\frac{dy}{dx}$ back in, then use the curve.	E03 · Practice 5, 6
A second derivative from a table	Find the formula for the second derivative with f, f', f'' first, then the numbers from the table.	E02 · Practice 7, 8

Example 01

Video · E01

If $f(x) = x\,e^{2x}$, then $f''(0) =$

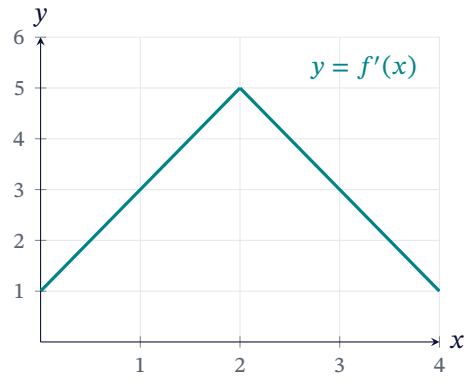
A. 1
B. 2
C. 4
D. 12

> Find f' first, then differentiate f'.

Example 02

Video · E02

The graph of f', made of line segments, is shown. Let $g(x) = f(x^2)$. Then $g''(1) =$



A. 4
B. 6
C. 8
D. 14

g' by the chain rule, then the product rule.

Example 03

Video · E03

Consider the curve $x^2 + 4y^2 = 8$.
(a) Show that $\frac{dy}{dx} = -\frac{x}{4y}$.
(b) Find $\frac{d^2y}{dx^2}$ at $(2, 1)$.

Implicit twice; put in $\frac{dy}{dx}$, then the point.

VIDEO COMPLETE Check your example answers at the end of this booklet.

3.6 / Example answers

Calculating Higher-Order Derivatives

E01 (C)

E02 (D)

E03 (a) shown; (b) $-\frac{1}{2}$