3.5 / Selecting Procedures for Calculating Derivatives

Choosing the rule

Watch the lesson ➤

Learning goal

Choose the first rule from the last operation, rewrite when it is shorter, and go inside one rule at a time.

Key ideas

01 Powers of x, added: term by term (rewrite $\sqrt{x}$, $\frac{5}{x}$ as powers first).

02 Times: the product rule.

03 Over, and you can't rewrite: the quotient rule.

04 An inside: the chain rule, ending with the inside's derivative.

05 x and y mixed, can't solve for y: implicit.

06 An inverse with no formula: find a with $f(a) = b$, then $g'(b) = \frac{1}{f'(a)}$.

07 $\tan^{-1}$, $\sin^{-1}$ of an inside: the formula, times u'.

08 The **last** operation picks the first rule. Then go inside, one rule at a time.

Remember

last operation
The operation done last when you compute the function's value; it picks the first rule.

Class notes

3.5 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The inside's derivative

Code C1

✗ $k(x) = (x^2 + 1)^5$: $k'(x) = 5(x^2 + 1)^4$, so $k'(1) = 80$.

✓ Times the inside's derivative $2x$: $k'(x) = 5(x^2+1)^4 \cdot 2x$, $k'(1) = 160$.

3.5 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
The last operation picks the first rule	Ask what a calculator does last: times -> product rule, divide -> quotient rule, a power or a function of something -> chain rule. Then go inside.	E02, E03, E04, E05, E06, E07, E08 · Practice 1, 3, 4, 5, 6, 7, 8, 10
Rewrite first, when that is shorter	A sum over x: split it. A number over a power: move the power up. A log of a product or a power: log properties ($x > 0$).	Practice 2
A rule inside a rule, shown step by step	Write the outer rule with its pieces named, find each piece (with the chain rule inside), then simplify to the given form.	Practice 9
Where is the x? Base, exponent, or nowhere	x in the base: power rule. x in the exponent: $a^x \ln a$. No x: a constant, 0.	E01 · Practice 11

Example 01

Video · E01

Let $f(x) = \frac{3}{x} + 3^x + e^3$. Find $f'(1)$.

Example 02

Video · E02

Let $g(x) = x^2 \ln(x^2 + 1)$. What is $g'(1)$?

A. 1
B. $2 \ln 2$
C. $2 \ln 2 + \frac{1}{2}$
D. $2 \ln 2 + 1$

> x^2 times $\ln(x^2 + 1)$, so the product rule, with the chain rule inside it.

Example 03

Video · E03

Let $h(x) = \dfrac{2x+1}{x^2+1}$. Find $h'(1)$.

A sum over a sum, so the quotient rule.

Example 04

Video · E04

Let $k(x) = (x^2+1)^5$. Find $k'(1)$.

An inside to a power, so the chain rule.

Example 05

Video · E05

The curve $x^3 + y^3 = 6xy$ passes through $(3, 3)$. Find $\dfrac{dy}{dx}$ at $(3, 3)$.

You can't solve for y, so implicit.

Example 06

Video · E06

Let $f(x) = x^5 + 2x - 1$, and let g be the inverse function of f. What is $g'(2)$?

A. $\frac{1}{7}$
B. $\frac{1}{82}$
C. 7
D. $\frac{1}{2}$

> Find a with $f(a) = 2$, then flip $f'(a)$.

Example 07

Video · E07

Let $y = \tan^{-1}(x^2)$. Find $\frac{dy}{dx}$ at $x = 1$.

> $\tan^{-1}$ of an inside, so the formula, times u'.

Example 08

Video · E08

p and q are differentiable; values are in the table.

Let $k(x) = \dfrac{p(q(x))}{x}$. Find $k'(1)$.

x	1	2
$p(x)$	4	5
$p'(x)$	-1	2
$q(x)$	2	1
$q'(x)$	4	3

VIDEO COMPLETE Check your example answers at the end of this booklet.

3.5 / Example answers

Selecting Procedures for Calculating Derivatives

E01 $f'(1) = -3 + 3\ln 3$

E02 (D)

E03 $-\frac{1}{2}$

E04 160

E05 -1

E06 (A)

E07 1

E08 3