3.4 / Practice workbook

Differentiating Inverse Trigonometric Functions

Practice

Problems 1–8 / Show your reasoning

1 Find the derivative of each function. (a) $y = \sin^{-1}\left(\frac{x}{2}\right)$ (b) $y = \tan^{-1}\left(x^3\right)$ (c) $y = \cos^{-1}(5x)$

2 Let $f(x) = \left(\tan^{-1} x\right)^2$. Find $f'(1)$.

3 If $y = \tan^{-1}\left(e^x\right)$, then $\frac{dy}{dx}$ at $x = \ln 2$ is

A. $\frac{1}{5}$
B. $\frac{2}{5}$
C. $\frac{2}{3}$
D. 2

4 If $y = \sin^{-1} x + \cos^{-1} x$, then $\frac{dy}{dx} =$

A. $\frac{2}{\sqrt{1-x^2}}$
B. $\frac{1}{\sqrt{1-x^2}}$
C. 0
D. $\frac{\pi}{2}$

5 Let $g(x) = f\left(\tan^{-1} x\right)$, where f is differentiable and $f'\left(\frac{\pi}{4}\right) = 6$. What is the value of $g'(1)$?

A. 3
B. $3\sqrt{2}$
C. 6
D. 12

3.4 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

A person stands x feet from a wall and looks at a mark on the wall 10 feet above eye level. The angle of elevation is $\theta(x) = \tan^{-1}\left(\frac{10}{x}\right)$ radians, for $x > 0$.

(a) Find $\theta'(10)$. Using correct units, explain its meaning.

(b) Show that $\theta'(x) < 0$ for all $x > 0$.

3.4 / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

The table gives values of a differentiable function f and its derivative. Let $h(x) = x\tan^{-1}(f(x))$.

x	1	2
$f(x)$	0	1
$f'(x)$	3	6

(a) Find $h'(2)$.

(b) Write an equation of the line tangent to the graph of h at $x = 2$.

8 Let $y = \tan^{-1}(3x)$. Write an equation of the line tangent to the graph at $x = \frac{1}{3}$.

3.4 / Answer key

Differentiating Inverse Trigonometric Functions

1 (a) $\dfrac{1}{\sqrt{4-x^2}}$; (b) $\dfrac{3x^2}{1+x^6}$; (c) $-\dfrac{5}{\sqrt{1-25x^2}}$

2 $f'(1) = 2 \cdot \dfrac{\pi}{4} \cdot \dfrac{1}{2} = \dfrac{\pi}{4}$

3 (B)

4 (C)

5 (A)

6 (a) $\theta'(10) = -\dfrac{1}{20}$ radian per foot: at 10 feet from the wall, the angle is decreasing at $\dfrac{1}{20}$ radian per foot; (b) $\theta'(x) = -\dfrac{10}{x^2+100} < 0$ for every $x > 0$

7 (a) $h'(2) = \tan^{-1} 1 + 2 \cdot \dfrac{6}{1+1^2} = \dfrac{\pi}{4} + 6$; (b) $y = \dfrac{\pi}{2} + \left(\dfrac{\pi}{4} + 6\right)(x-2)$

8 $y = \dfrac{\pi}{4} + \dfrac{3}{2}\left(x - \dfrac{1}{3}\right)$