Lecture Notes · Unit 3 · Lecture 6 AP CED 3.4

Where Did the Trig Go? Inverse Trig Derivatives (AP Calc AB 3.4)

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

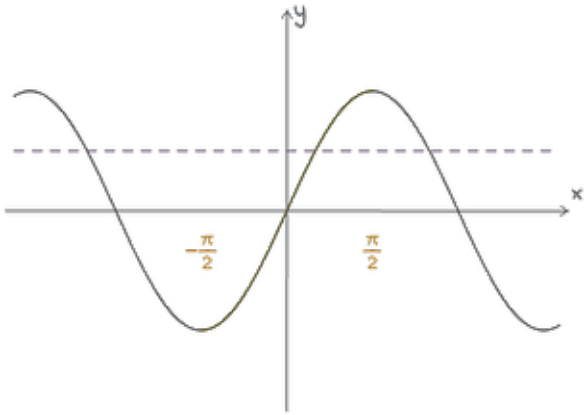
Inverse trig

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

Today → where did the trig go?

$\sin^{-1}\frac{1}{2}$ → angle

$\sin\frac{\pi}{6} = \frac{1}{2} \rightarrow \frac{\pi}{6}$

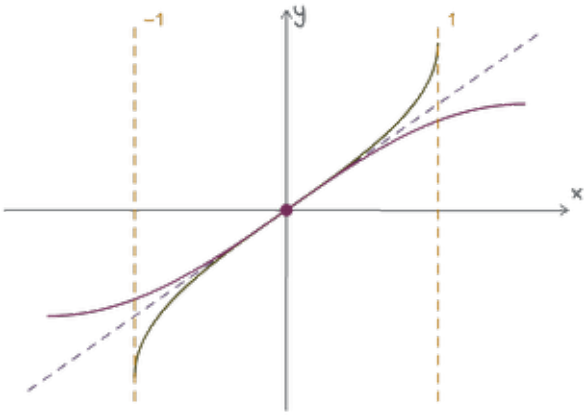


$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

each height once → one-to-one

$\sin^{-1}x = \arcsin x$

f^{-1}, $\sin^{-1}x \neq \frac{1}{\sin x}$

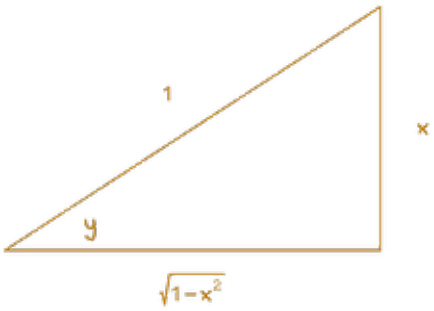


$y = \sin^{-1}x$

flip → slope m becomes $\frac{1}{m}$

$x = 0$ → slope 1

$x = \pm 1$ → vertical tangent



$y = \sin^{-1}x \rightarrow \sin y = x$

$e^y = x$

$$\cos y \cdot \frac{dy}{dx} = 1$$

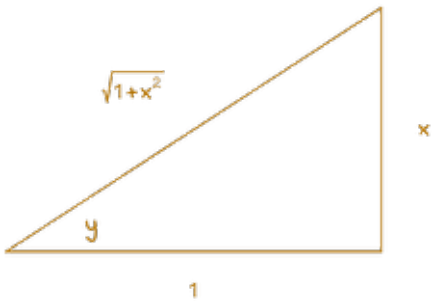
$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$\cos y$, not $\cos x$

$\sin y = \frac{x}{1}$

$$\cos y = \sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$



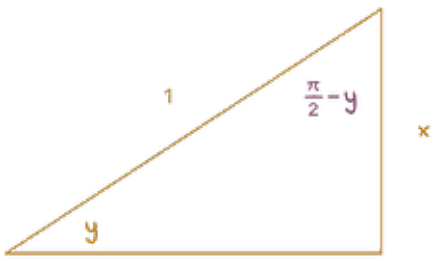
$$y = \tan^{-1} x,\ \tan y = x$$

$$\sec^2 y \cdot \frac{dy}{dx} = 1$$

$$y' = \frac{1}{\sec^2 y} = \cos^2 y$$

$$\cos^2 y = \frac{1}{1+x^2}$$

$$\tan^{-1} x \to \frac{1}{1+x^2}$$



$$\cos\left(\frac{\pi}{2} - y\right) = x$$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

holds on $[-1,1] \to \cos^{-1} x$ in $[0,\pi]$

$$\frac{1}{\sqrt{1-x^2}} + (\cos^{-1} x)' = 0$$

$$(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$$

3 functions · 2 forms: x (left), u(x) (right)

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\sin^{-1} u = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\cos^{-1} u = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\tan^{-1} u = \frac{u'}{1+u^2}$$

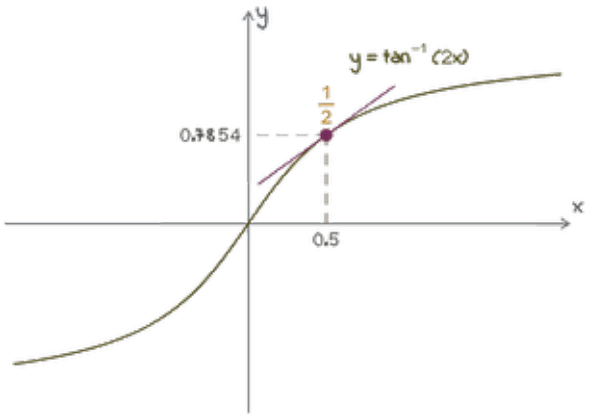
$u = 3x \to u' = 3$

$$\frac{d}{dx}\sin^{-1}(3x) = \frac{3}{\sqrt{1-(3x)^2}} = \frac{3}{\sqrt{1-9x^2}}$$

PROBLEM 1 OF 3

Let $y = \tan^{-1}(2x)$.

Write an equation of the tangent line to the graph at $x = \frac{1}{2}$.



Solution

Point → slope → chain

$$y\left(\frac{1}{2}\right) = \tan^{-1} 1$$

$$= \frac{\pi}{4} \rightarrow \left(\frac{1}{2}, \frac{\pi}{4}\right)$$

$u = 2x,\ u' = 2$

$$y' = \frac{2}{1+4x^2}$$

$$y'\left(\frac{1}{2}\right) = \frac{2}{2} = 1$$

$$y = \frac{\pi}{4} + 1\left(x - \frac{1}{2}\right)$$

Workbook

where the slope goes wrong

X Common mistake

$$y' = \frac{1}{1+4x^2}$$

$$y'\left(\frac{1}{2}\right) = \frac{1}{2}$$

✓ Do this

$$y' = \frac{2}{1+4x^2} \rightarrow 1$$

$u' \uparrow$

PROBLEM 2 OF 3

If $f(x) = x\sin^{-1} x$, then $f'\left(\frac{1}{2}\right) =$

(A) $\frac{\pi}{6}$ (B) $\frac{\sqrt{3}}{3}$ (C) $\frac{\pi}{6} + \frac{\sqrt{5}}{5}$ (D) $\frac{\pi}{6} + \frac{\sqrt{3}}{3}$

$$(uv)' = u'v + uv'$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

Solution

Product → $x = \frac{1}{2}$

$$f'(x) = \sin^{-1} x + x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \rightarrow \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{3}$$

Workbook

PROBLEM 3 OF 3

The table gives values of a differentiable function f and its derivative.

(a) Let $h(x) = \tan^{-1}(f(x))$. Find $h'(3)$.

(b) Let $k(x) = \sin^{-1}(f(x))$. Find $k'(2)$.

x	1	2	3
$f(x)$	2	0	1
$f'(x)$	−1	3	4

Solution

(a)

$u = f(x) \rightarrow$ table $x = 3$

$$h'(3) = \frac{f'(3)}{1 + f(3)^2}$$

$f(3) = 1,\ f'(3) = 4$

$$= \frac{4}{1 + 1^2} = 2$$

Solution

x	1	2	3
$f(x)$	2	0	1
$f'(x)$	−1	3	4

(b)

$u = f(x) \rightarrow x = 2$

$$\frac{f'(2)}{\sqrt{1 - f(2)^2}}$$

$f(2) = 0\ f'(2) = 3$

$$\frac{3}{\sqrt{1}} = 3$$

Workbook

Next → 3.5 choosing the right rule