3.3 / Practice workbook

Differentiating Inverse Functions

Practice

Problems 1–6 / Show your reasoning

1 Let $f(x) = x^5 + 3x - 2$, and let g be the inverse function of f. (a) Find $g'(2)$. (b) Find $g'(-2)$.

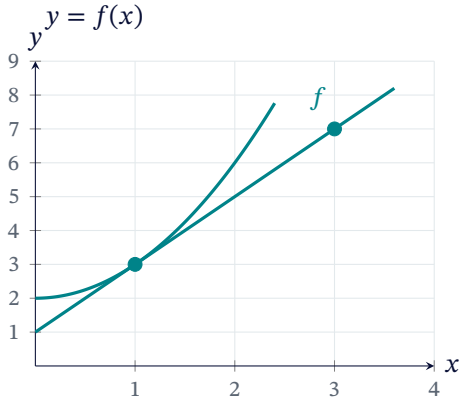
2 Let $f(x) = 2x + \ln x$ for $x > 0$, and let g be the inverse function of f. (a) Find $g(2)$. (b) Find $g'(2)$. (c) Write an equation of the line tangent to the graph of g at $x = 2$.

3 The function f is differentiable and has an inverse function g. Values of f and f' are given in the table. What is the value of $g'(5)$?

x	2	5
$f(x)$	5	8
$f'(x)$	2	4

A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. 2
D. 4

4 The graph of f and its tangent line at $x = 1$ are shown. The line passes through $(1, 3)$ and $(3, 7)$. If g is the inverse of f, what is $g'(3)$?



A. $-\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. 2

5 Let f be a differentiable function with an inverse function g. If $f(4) = 7$ and $f'(4) = 0$, which of the following is true?

A. $g'(7) = 0$

B. $g'(7) = \frac{1}{7}$

C. $g(7) = 0$

D. The graph of g has a vertical tangent line at $(7, 4)$.

3.3 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

A pump removes water from a tank. The total amount of water removed in the first t minutes is $f(t) = t^2 + 5t$ gallons, for $t \geq 0$. The function g is the inverse function of f.

(a) Find $g(50)$. Explain its meaning, with units.

(b) Find $g'(50)$. Using correct units, explain its meaning.

3.3 / Answer key

Differentiating Inverse Functions

1 (a) $f(1) = 2$, so $g'(2) = \dfrac{1}{f'(1)} = \dfrac{1}{8}$; (b) $f(0) = -2$, so $g'(-2) = \dfrac{1}{f'(0)} = \dfrac{1}{3}$

2 (a) $f(1) = 2 + \ln 1 = 2$, so $g(2) = 1$; (b) $f'(x) = 2 + \dfrac{1}{x}$, $f'(1) = 3$, so $g'(2) = \dfrac{1}{3}$; (c) $y = 1 + \dfrac{1}{3}(x - 2)$

3 (B)

4 (C)

5 (D)

6 (a) $f(5) = 25 + 25 = 50$, so $g(50) = 5$ minutes: it takes 5 minutes to remove 50 gallons; (b) $g'(50) = \dfrac{1}{f'(5)} = \dfrac{1}{15}$ minute per gallon: when 50 gallons have been removed, the time is increasing at $\dfrac{1}{15}$ minute per gallon