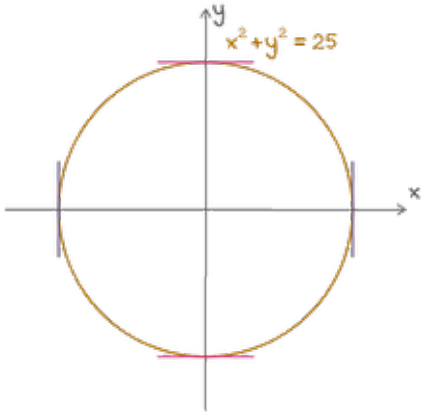
Lecture Notes · Unit 3 · Lecture 4 AP CED 3.2b

Tangents to implicit curves

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: WHERE IS A CURVE FLAT, OR STRAIGHT UP?

Horizontal & vertical tangents AP Calculus 3.2



$x^2+y^2=25$, radius 5

Flat where? Straight up where?

$x^2-xy+y^2=27$ → slope formula alone?

$2x\,dx+2y\,dy=0 \rightarrow \frac{dy}{dx}=-\frac{x}{y} \rightarrow \frac{\text{numerator}}{\text{denominator}}$

Flat → slope 0 → numerator = 0, bottom ≠ 0

Straight up → undef. → denominator = 0, top ≠ 0

Both 0? $\frac{0}{0}$ → tells nothing

Numerator $x=0 \rightarrow y^2=25 \rightarrow (0,\pm5)$; $y\neq 0$

$y=0 \rightarrow x^2=25 \rightarrow (\pm5,0)$; $-x\neq 0$

→ horizontal / vertical tangents

No point yet → back to the curve

PROBLEM 1 OF 2

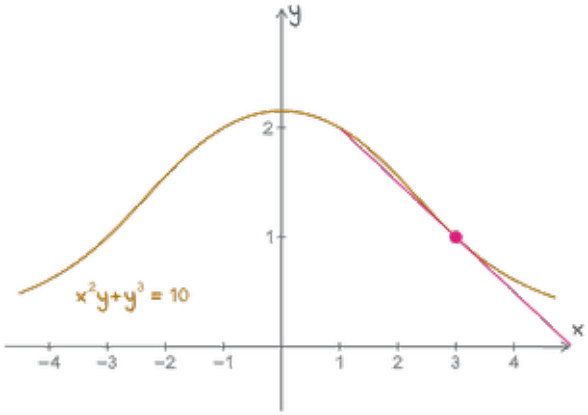
A tangent line, y not solved

A curve is given by $x^2y+y^3=10$.

(a) Find $\frac{dy}{dx}$ at the point $(3, 1)$.

(b) Write an equation of the tangent line to the curve at $(3, 1)$.

(c) Is there a point on the curve where the tangent line is vertical? Give a reason.



Solution (b) A point and a slope

(a)

$$\frac{-2(3)(1)}{3^2+3(1)^2}=\frac{-6}{12}=-\frac{1}{2}$$

(b): your turn.

(b)

Plan: point $(3, 1)$, slope $-\frac{1}{2}$

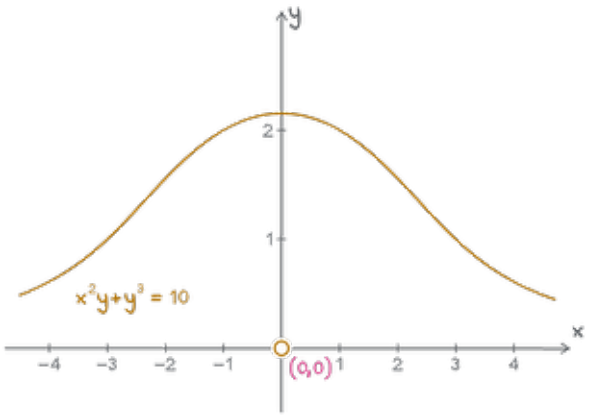
$$y=1-\frac{1}{2}(x-3)$$

Point-slope form → you can stop here

Same slope at (3, 1).

Solution

(c) Vertical: denominator = 0, then back to the curve



Plan: vertical → $x^2 + 3y^2 = 0$ → on the curve?

(c)

Squares ≥ 0 → only $x = y = 0$

one candidate: (0, 0)

Yes, at (0, 0)? on the curve?

$0^2 \cdot 0 + 0^3 = 0 \neq 10$ → not on the curve

So no vertical tangent line.

AP 2023: none, why → 2 pts;
vertical + check → 3 pts

Workbook: none → say why

PROBLEM 2 OF 2

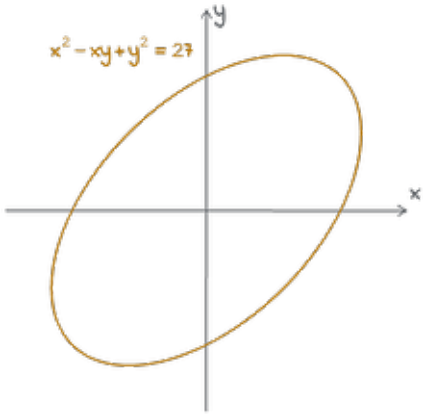
Horizontal and vertical tangents

A curve is given by $x^2 - xy + y^2 = 27$.

(a) Show that $\frac{dy}{dx} = \frac{2x - y}{x - 2y}$.

(b) Find all points with a horizontal tangent.

(c) Find all points with a vertical tangent.



Solution (a) Show that: both sides

(a)

Plan: both sides → toward the given form

$$xy \to (fg)' = f'g + fg' \to 1 \cdot y + x\frac{dy}{dx}$$

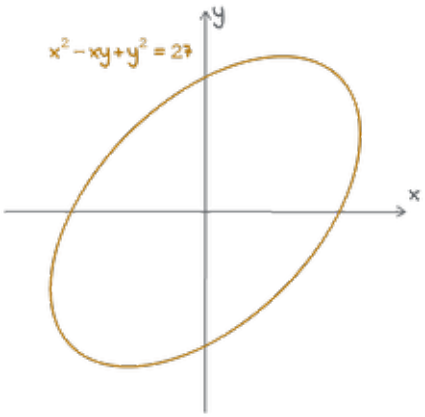
$$2x - \left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$\to 2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

Solution (a) Show that: match the given form



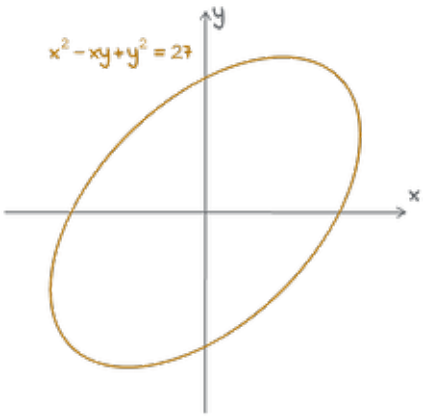
Given: $\frac{2x-y}{x-2y}$ → signs backwards?

$$\frac{y-2x}{2y-x}\cdot\frac{-1}{-1}=\frac{2x-y}{x-2y}$$

Given formula → (b), (c) start from it

(b) horizontal: your turn.

Solution (b) Numerator = 0, then back to the curve



Plan: $2x-y=0$ → the curve

(b)

$2x-y=0 \rightarrow y=2x$

A whole line ≠ a point → the curve

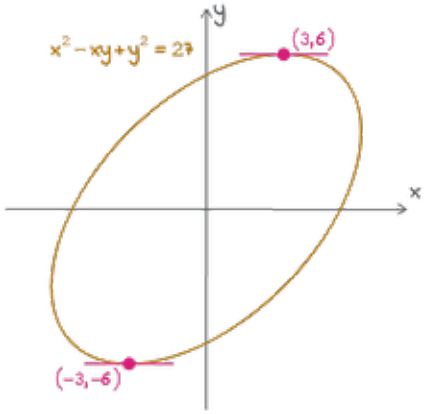
$x^2-x(2x)+(2x)^2=27$

$x^2-2x^2+4x^2=3x^2=27$

$x^2=9 \rightarrow x=3$ or -3

Your turn: the y-values.

Solution (b) The y-values, then the denominator



Plan: y-values → denominator

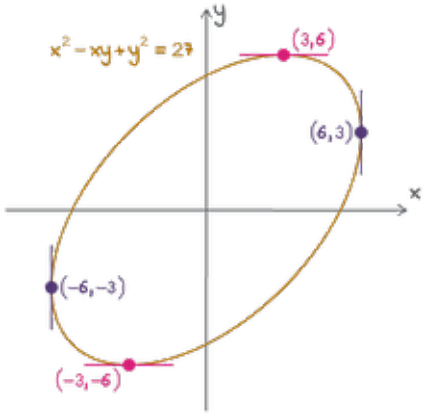
$x=3 \rightarrow y=6, x=-3 \rightarrow y=-6$

$x-2y \rightarrow \pm(3-12)=\mp 9 \neq 0$

(3,6), (−3,−6)

(c) vertical: your turn.

Solution (c) Denominator = 0, then back to the curve



Plan: $x-2y=0$ → the curve

(c)

$x - 2y = 0 \rightarrow x = 2y$

$(2y)^2 - (2y)y + y^2 = 27$

$4y^2 - 2y^2 + y^2 = 3y^2 = 27$

$y = 3$ or -3

$y = 3 \rightarrow x = 6, y = -3 \rightarrow x = -6$

$2x - y \rightarrow \pm(12 - 3) = \pm 9 \neq 0$

$(6, 3), (-6, -3)$

Workbook: flat, vertical → back to the curve

WRAP-UP

Flat or straight up?

Not alone: numerator 0 → horizontal; denominator 0 → vertical

Tangent line: point + slope → point-slope

Condition → a whole line, a candidate → back to the curve

Not on the curve → none, and that is the reason

Next → 3.3: derivatives of inverse functions

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.