# 3.2b / Tangent Lines to Implicit Curves

Watch the lesson ➤

*Implicit differentiation · Part 2*

**Learning goal**
Use dy/dx of an implicit curve for tangent lines, horizontal and vertical tangents, and say with a reason when there is no such point.

## Key ideas

01 A tangent line at a point $(a, b)$ of a curve: check that the point is on the curve, then the slope is $\frac{dy}{dx}$ at $(a, b)$: $y = b + m(x - a)$.

02 Horizontal tangent: top $= 0$ (bottom $\neq 0$). Vertical: bottom $= 0$ (top $\neq 0$). Then use the **curve** to find the point. Circle: top $x = 0$ gives $(0, \pm 5)$; bottom $y = 0$ gives $(\pm 5, 0)$.

03 A condition such as $y = 2x$ is not an answer yet: put it **into the curve** to find the points. If no point of the curve fits, the answer is **no such point**, with that reason.

## Remember

**vertical tangent**
Where the bottom of $\frac{dy}{dx}$ is 0 and the top is not.

**horizontal tangent**
Where the top of $\frac{dy}{dx}$ is 0 and the bottom is not.

Class notes

# 3.2b / Write it clearly

*What earns the point on the AP Exam, and the mistakes that lose it.*

## Common mistakes

The mistake code is used in the answers and in My Mistakes

### 01 A candidate that is not on the curve

Code I2

✗ $x^2 + 3y^2 = 0$ gives $(0, 0)$, so $x^2y + y^3 = 10$ has a vertical tangent at $(0, 0)$.

✓ Check it on the curve: $0 + 0 \neq 10$. $(0, 0)$ is not on the curve, so there is no vertical tangent.

# 3.2b / Guided examples

*Watch, pause, and write the reasoning. Full solutions are in the video.*

## Problem types

Read the problem, name the type, then use its method

| Type | Method | Problems |
|---|---|---|
| Tangent line to a curve given implicitly | The point is given (check it is on the curve); the slope is $\frac{dy}{dx}$ at that point. | E01 · Practice 1 |
| Horizontal and vertical tangents | Horizontal: top $= 0$, bottom $\neq 0$. Vertical: bottom $= 0$, top $\neq 0$. Then use the curve to find the points. | E02 · Practice 2, 3, 4, 5 |

### Example 01

Video · E01

A curve is given by $x^2y + y^3 = 10$.
(a) Find $\frac{dy}{dx}$ at the point $(3, 1)$.
(b) Write an equation of the tangent line to the curve at $(3, 1)$.
(c) Is there a point on the curve where the tangent line is vertical? Give a reason.

> Check the point is on the curve. Then differentiate both sides.

### Example 02

Video · E02

A curve is given by $x^2 - xy + y^2 = 27$.
(a) Show that $\frac{dy}{dx} = \frac{2x - y}{x - 2y}$.
(b) Find all points with a horizontal tangent.
(c) Find all points with a vertical tangent.

> Horizontal means the top is 0; vertical means the bottom is 0. Then use the curve.

**VIDEO COMPLETE** Check your example answers at the end of this booklet.

# 3.2b / Example answers

*Tangent Lines to Implicit Curves*

E01 (a) $-\frac{1}{2}$; (b) $y = 1 - \frac{1}{2}(x - 3)$; (c) no

E02 (a) shown; (b) $(3, 6)$ and $(-3, -6)$; (c) $(6, 3)$ and $(-6, -3)$