3.2a / Implicit Differentiation: Finding dy/dx

Watch the lesson ➤

Implicit differentiation · Part 1

Learning goal

Find dy/dx for a curve that is not solved for y, and its value at a point, with every y term getting its dy/dx.

Key ideas

01 $x^2+y^2=25$ is not a function. The top half $y=\sqrt{25-x^2}$ gives $y'=\dfrac{-x}{\sqrt{25-x^2}}$, $-\frac{3}{4}$ at $x=3$. It works, but we had to **solve** for y first.

02 Differentiate both sides instead, each letter with its own d: $x^2 \to 2x\,dx$, $y^2 \to 2y\,dy$; divide by dx: $2y\dfrac{dy}{dx}$ (outside u^2, inside y: the chain rule). $2x+2y\dfrac{dy}{dx}=0$ (since $\frac{d}{dx}25=0$), so $\dfrac{dy}{dx}=-\dfrac{x}{y}$: $-\frac{3}{4}$ at $(3,4)$, $\frac{3}{4}$ at $(3,-4)$. That is **implicit** differentiation.

03 Each y-term gets $\dfrac{dy}{dx}$: $\dfrac{d}{dx}(xy)=y+x\dfrac{dy}{dx}$, $\dfrac{d}{dx}\sin y=\cos y\dfrac{dy}{dx}$, $\dfrac{d}{dx}e^y=e^y\dfrac{dy}{dx}$; $\dfrac{d}{dx}x^3=3x^2$ has none.

04 Keep the $=0$, move the $\dfrac{dy}{dx}$ terms to one side, factor out $\dfrac{dy}{dx}$, then divide.

Remember

implicit differentiation
Differentiating both sides of an equation in x and y, with $\frac{dy}{dx}$ on every y-term.

Class notes

3.2a / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 A product with y needs the product rule

Code I3

✗ $\frac{d}{dx}(x^2y) = 2x\frac{dy}{dx}$.

✓ x^2y is a product: $2xy + x^2\frac{dy}{dx}$.

02 y is not a constant

Code I1

✗ $\frac{d}{dx}(y^3) = 0$.

✓ y depends on x: $\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$.

3.2a / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
$\frac{dy}{dx}$ of a curve and its value at a point	Differentiate both sides; every y term gets a $\frac{dy}{dx}$. Then solve for $\frac{dy}{dx}$, or plug in the point first.	E01, E02 · Practice 1, 2, 3, 4, 5, 6

Example 01

Video · E01

A curve is given by $x^2y + y^3 = 10$.
Find $\frac{dy}{dx}$ at the point $(3, 1)$.

> Check the point is on the curve. Then differentiate both sides; every y term gets dy/dx.

Example 02

Video · E02

If $e^y + xy = e$, what is the value of $\frac{dy}{dx}$ at the point $(0, 1)$?

> Both sides, then plug in $(0, 1)$.

A. $-\frac{1}{e}$
B. $-e$
C. $\frac{1}{e}$
D. 0

VIDEO COMPLETE Check your example answers at the end of this booklet.

3.2a / Example answers

Implicit Differentiation: Finding dy/dx

E01 $-\frac{1}{2}$

E02 (A)