3.1b / Practice workbook

The Chain Rule with Tables and Graphs

Practice

Problems 1–9 / Show your reasoning

1 Values of f, f', g, and g' are given in the table. Let $h(x) = f(g(x))$ and $k(x) = g(f(x))$. (a) Find $h'(2)$. (b) Find $k'(3)$.

x	1	2	3	4
$f(x)$	3	4	1	2
$f'(x)$	-2	5	3	-1
$g(x)$	2	4	3	1
$g'(x)$	6	-3	2	4

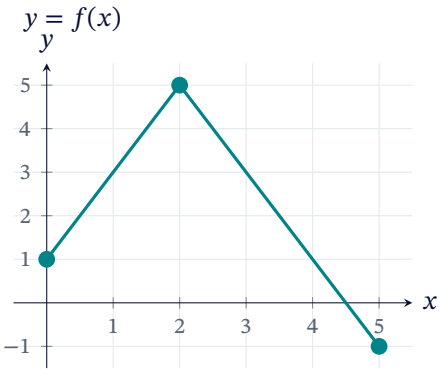
2 Values of f, f', g, and g' are given in the table. Let $p(x) = [f(x)]^3$ and $r(x) = f(2x)$. (a) Find $p'(1)$. (b) Find $r'(2)$.

x	1	2	3	4
$f(x)$	3	4	1	2
$f'(x)$	-2	5	3	-1
$g(x)$	2	4	3	1
$g'(x)$	6	-3	2	4

3 Values of f, f', g, and g' are given in the table. Let $m(x) = \ln(g(x))$. Write an equation of the line tangent to the graph of m at $x = 3$.

x	1	2	3	4
$f(x)$	3	4	1	2
$f'(x)$	-2	5	3	-1
$g(x)$	2	4	3	1
$g'(x)$	6	-3	2	4

4 The graph of f is made of line segments from $(0, 1)$ to $(2, 5)$ and from $(2, 5)$ to $(5, -1)$. Let $h(x) = [f(x)]^2$. What is $h'(3)$?



A. -12
B. -4
C. 4
D. 6

5 The functions f and g are differentiable, with $g(1) = 2$, $g'(1) = 3$, $f'(1) = 4$, $f'(2) = -1$, and $f(2) = 5$. If $h(x) = f(g(x))$, what is $h'(1)$?

A. -3
B. -1
C. 12
D. 15

6 Values of f and f' are given in the table. If $h(x) = f(f(x))$, what is $h'(1)$?

x	1	2	3	4
$f(x)$	3	4	1	2
$f'(x)$	-2	5	3	-1
$g(x)$	2	4	3	1
$g'(x)$	6	-3	2	4

A. -6
B. 3
C. 4
D. 9

3.1b / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

The number of fish in a lake is $F(T)$, where T is the water temperature in degrees Celsius. The water temperature is $T(t)$ degrees Celsius on day t; values are in the table. Also $F'(15) = -50$ and $F'(20) = -80$. Let $N(t) = F(T(t))$.

t (days)	5	10
$T(t)$	15	20
$T'(t)$	0.8	1.2

(a) Find $N'(5)$. Indicate units of measure.

(b) Explain the meaning of $N'(5)$ in context.

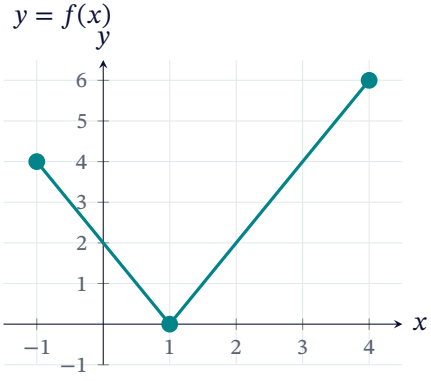
(c) Is the number of fish increasing or decreasing on day 10? Give a reason.

3.1b / AP-style free response

Problem 8 ➤ */ Show the work and the reasons a grader looks for.*

The graph of f is made of line segments from $(-1, 4)$ to $(1, 0)$ and from $(1, 0)$ to $(4, 6)$. Let $k(x) = \sqrt{f(x)}$ and $h(x) = f(2x + 1)$.

$y = f(x)$



(a) Find $k'(3)$.

(b) Find $h'(1)$.

(c) Is h differentiable at $x = 0$? Explain.

9 Values of f, f', g, and g' are given in the table. Let $w(x) = x^2 g(2x)$. Find $w'(1)$.

x	1	2	3	4
$f(x)$	3	4	1	2
$f'(x)$	-2	5	3	-1
$g(x)$	2	4	3	1
$g'(x)$	6	-3	2	4

3.1b / Answer key

The Chain Rule with Tables and Graphs

1 (a) $h'(2) = f'(g(2)) \cdot g'(2) = f'(4)(-3) = (-1)(-3) = 3$; (b) $k'(3) = g'(f(3)) \cdot f'(3) = g'(1)(3) = (6)(3) = 18$

2 (a) $p'(1) = 3[f(1)]^2 f'(1) = 3(9)(-2) = -54$; (b) $r'(2) = f'(4) \cdot 2 = (-1)(2) = -2$

3 $y = \ln 3 + \frac{2}{3}(x - 3)$

4 (A)

5 (A)

6 (A)

7 (a) $N'(5) = F'(T(5)) \cdot T'(5) = F'(15)(0.8) = -40$ fish per day; (b) on day 5, the number of fish in the lake is decreasing at a rate of 40 fish per day; (c) decreasing, because $N'(10) = F'(20)(1.2) = -96 < 0$

8 (a) $k'(3) = \frac{f'(3)}{2\sqrt{f(3)}} = \frac{2}{2 \cdot 2} = \frac{1}{2}$; (b) $h'(1) = f'(3) \cdot 2 = 4$; (c) no: the inside value is $2(0) + 1 = 1$, a corner of f; the slopes of h are $(-2)(2) = -4$ on the left and $(2)(2) = 4$ on the right

9 2