Lecture Notes · Unit 3 · Lecture 2 AP CED 3.1b

Chain Rule: Tables and Graphs

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: NO FORMULA, JUST A TABLE OR A GRAPH



Chain Rule: Tables and Graphs AP Calculus 3.1

No formulas → table or graph

$h(x) = f(g(x)) \rightarrow h'(2)$?

See 2 → look up $f'(2)$?

Today: do we need $f'(2)$?

value or slope · where f' is read

INSIDE FIRST: THE BATON, AND THE ORDER



Inside first, then the order

Inside first

$h'(x) = f'(g(x))g'(x)$

$x = 2 \rightarrow$?

$x = 2 \rightarrow g \rightarrow g(2) \rightarrow f'(g(2))$

$h'(2) = f'(g(2))g'(2) \rightarrow g(2)$ first

$g(2) = 5 \rightarrow f'(5)$, not $f'(2)$

$g(2)$ = the inside value

The order matters

$f(g(x)) \neq g(f(x))$ usually

$f(g(x)) \rightarrow f'(g(x))g'(x)$

$g(f(x)) \rightarrow g'(f(x))f'(x)$

Same letters, leftmost → the outside

$f(f(x)) \rightarrow f'(f(x))f'(x)$

$f(2) = 3 \rightarrow f'(3) \cdot f'(2)$, not $[f'(2)]^2$

LOOK-ALIKES: FIND THE INSIDE



Look-alikes

Look-alikes

$f(x^2)$ vs $[f(x)]^2$ → same? no

$\frac{d}{dx} f(x^2) \rightarrow x^2$ inside

$= f'(x^2) \cdot 2x$

$\frac{d}{dx}[f(x)]^2 \rightarrow$ square outside

$= 2f(x) \cdot f'(x)$

$\frac{d}{dx} f(3x) \rightarrow 3x$ inside

$= f'(3x) \cdot 3$

$\frac{d}{dx} 3f(x) \rightarrow$ no inside

$= 3f'(x)$

Twins → not related

1 question: inside?

STRUCTURE FIRST (AND HOW AP SCORES IT)



Structure first, then the numbers

Structure first, then the numbers

$h'(1) = f'(g(1)) \cdot g'(1)$

□ · □ → then fill the boxes

→ the structure line

2023 AP → structure, values scored separately

2026 AP → no structure, no answer point

Letters first, numbers second

PROBLEM 1 OF 3

A composite from a table

The functions f and g are differentiable; the table gives values.

(a) Let $h(x)=f(g(x))$. Find $h'(1)$.

(b) Let $k(x)=g(f(x))$. Find $k'(2)$.

(c) Write an equation of the tangent line to the graph of h at $x=1$.

x	1	2	3	4
$f(x)$	4	3	2	−1
$f'(x)$	−2	6	7	3
$g(x)$	3	1	4	2
$g'(x)$	5	−1	2	$\frac{1}{2}$

Solution (a) Structure first, inside value first

(a)

Plan: structure → inside value

$h'(1)=f'(g(1))g'(1)$

Inside: $x=1$, g row → $g(1)=3$

$=f'(3)\cdot g'(1)$

$=7\cdot 5=35$

Careful f' at the inside value

x	1	2	3	4
$f(x)$	4	3	2	−1
$f'(x)$	−2	6	7	3
$g(x)$	3	1	4	2
$g'(x)$	5	−1	2	$\frac{1}{2}$

✗ Common mistake

$h'(1)=f'(1)\cdot g'(1)$

$=(-2)(5)=-10$

✓ Do this

f' at $g(1)=3$, not at 1.

$h'(1)=f'(3)\cdot 5=35$

Solution (b) The order flips

x	1	2	3	4
$f(x)$	4	3	2	−1
$f'(x)$	−2	6	7	3
$g(x)$	3	1	4	2
$g'(x)$	5	−1	2	$\frac{1}{2}$

(b)

Plan: $g(f(x))$ → g outside

$k'(2)=g'(f(2))f'(2)$

Inside: $x=2$, f row → $f(2)=3$

$=g'(3)\cdot f'(2)$

$=2\cdot 6=12$

g on the outside → g' first

(c) → slope ✓

Solution (c) A point from values, a slope from (a)

x	1	2	3	4
$f(x)$	4	3	2	−1
$f'(x)$	−2	6	7	3
$g(x)$	3	1	4	2
$g'(x)$	5	−1	2	$\frac{1}{2}$

(c)

Plan: point $h(1)$, slope $h'(1)$

$h(1)=f(g(1))=f(3)=2$

Slope → $h'(1)=35$ from (a)

$(1,2)$, $m=35$ → $y=2+35(x-1)$

PROBLEM 2 OF 3

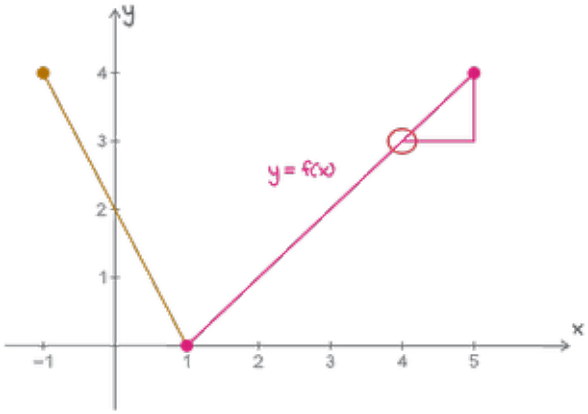
A composite from a graph

The graph of f is made of two line segments.

Let $g(x)=x^2$ and $h(x)=f(g(x))$.

(a) Find $h'(2)$.

(b) Find $h'(0)$.



Solution (a) Inside value, then the segment's slope

(a)

Plan: $h'(x)=f'(g(x))g'(x)$, $g'(x)=2x$

Graph: $f(a)$ = height, $f'(a)$ = slope

$h'(2)=f'(g(2))g'(2)$

Inside: $g(2)=2^2=4$

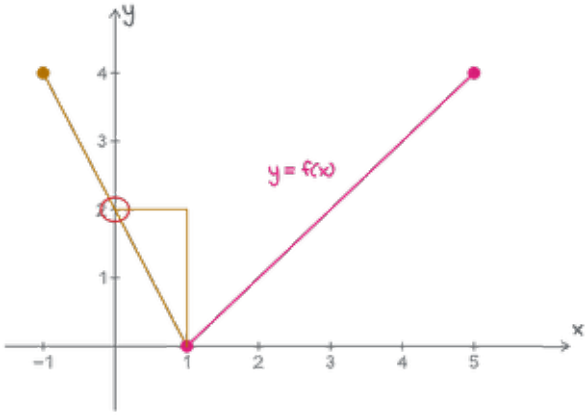
Height 3 X → right segment → $f'(4)=1$

$g'(2)=2\cdot 2=4$

$=f'(4)\cdot g'(2)=1\cdot 4=4$

(b) → inside value first

Solution (b) A flat inside



(b)

Plan: new $g(0)$

$h'(0)=f'(g(0))g'(0)$

$g(0)=0$ → left segment, $f'(0)=-2$

-2? Times $g'(0)$!

$g'(0)=2\cdot 0=0$

$h'(0)=(-2)\cdot 0=0$

Flat inside, $g'(0)=0$ → product 0

PROBLEM 3 OF 3

A product with a composite inside

Let $q(x)=x\,g(x^2)$, with g from the table. What is $q'(2)$?

(A) 3 (B) 4 (C) 6 (D) −6

x	1	2	3	4
$f(x)$	4	3	2	−1
$f'(x)$	−2	6	7	3
$g(x)$	3	1	4	2
$g'(x)$	5	−1	2	$\frac{1}{2}$

Solution Product rule first, a chain inside

Plan: product $\times g(x^2)$, chain inside

$(uv)' = u'v + uv'$

$q'(x) = 1 \cdot g(x^2) + x \cdot g'(x^2) \cdot 2x$

Your turn: plug in $x = 2$

Solution Plug in $x = 2$, then the table

x	1	2	3	4
$f(x)$	4	3	2	−1
$f'(x)$	−2	6	7	3
$g(x)$	3	1	4	2
$g'(x)$	5	−1	2	$\frac{1}{2}$

Plan: $x = 2 \to x^2 = 4$, $2x = 4$

$q'(2) = g(4) + 2 \cdot g'(4) \cdot 4$

$g(4) = 2, g'(4) = \frac{1}{2}$

$= 2 + 2 \cdot \frac{1}{2} \cdot 4 = 2 + 4 = 6$

So (C)

WRAP-UP: BACK TO TODAY'S QUESTION

For $h'(2)$, $f'(2)$?

only if $g(2) = 2$

$h'(2) = f'(g(2)) \cdot g'(2) \to f'$ at the inside value

1. Structure line, letters first
2. The inside value before any slope; watch the order
3. A product? Product first, chain inside

Next → 3.2: when you can't solve for y