3.1b / The Chain Rule with Tables and Graphs

Watch the lesson ➤

The chain rule with tables and graphs · Part 2

Learning goal

Use the chain rule with tables and graphs: find the inside value first, then read the outside's slope there.

Key ideas

01 $h(x) = f(g(x))$: for $h'(2)$, do we need $f'(2)$? No: $x = 2 \to g(2) \to f'(g(2))$. So $h'(2) = f'(g(2)) \cdot g'(2)$; find $g(2)$ **first**. If $g(2) = 5$, read $f'(5)$, not $f'(2)$.

02 $\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$ and $\frac{d}{dx} g(f(x)) = g'(f(x))\, f'(x)$: the leftmost letter is the **outside**.

03 Look-alikes: $\frac{d}{dx} f(x^2) = f'(x^2) \cdot 2x$, $\frac{d}{dx}[f(x)]^2 = 2f(x) \cdot f'(x)$, $\frac{d}{dx} f(3x) = f'(3x) \cdot 3$, $\frac{d}{dx} 3f(x) = 3f'(x)$. First find the inside.

04 A zero inside slope: with $g(x) = x^2$, $h'(0) = f'(g(0)) \cdot g'(0) = f'(0) \cdot 0 = 0$. A **zero** inside slope makes the product 0.

05 Write $\square \cdot \square$ first, then fill the boxes: the **structure** line can earn a point by itself.

Remember

inside value
The number $g(a)$ where the outside's derivative is read: $h'(a) = f'(g(a))\, g'(a)$.

structure line
The chain rule written with letters, $f'(g(1)) \cdot g'(1)$, before the numbers go in.

Class notes

3.1b / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The outside's slope at the inside value

Code C2

✗ $h = f(g(x))$: $h'(1) = f'(1) \cdot g'(1) = (-2)(5) = -10$.

✓ f' at $g(1) = 3$, not at 1: $h'(1) = f'(3) \cdot g'(1) = 7 \cdot 5 = 35$.

3.1b / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
$h'(a)$ of a composite from a table	The structure $h'(a) = f'(g(a)) \cdot g'(a)$ first, then find $g(a)$ first, then read f' at that number.	E01 · Practice 1, 5, 6
Look-alikes with a table ($[f(x)]^n$, $f(kx)$, $\ln g(x)$)	Find the inside first: $[f(x)]^n$ has inside $f(x)$; $f(kx)$ has inside kx.	E03 · Practice 2, 3, 9
The chain rule read from a graph	A value is a height; a derivative is the slope of the segment through that point.	E02 · Practice 4, 8
A composite rate in context, with units	Units multiply: (fish per degree) times (degrees per day) = fish per day.	Practice 7

Example 01

Video · E01

f and g are differentiable; values are in the table.
(a) Let $h(x) = f(g(x))$. Find $h'(1)$.
(b) Let $k(x) = g(f(x))$. Find $k'(2)$.
(c) Write an equation of the tangent line to the graph of h at $x = 1$.

x	1	2	3	4
$f(x)$	4	3	2	-1
$f'(x)$	-2	6	7	3
$g(x)$	3	1	4	2
$g'(x)$	5	-1	2	$\frac{1}{2}$

> The structure first, the inside value first.

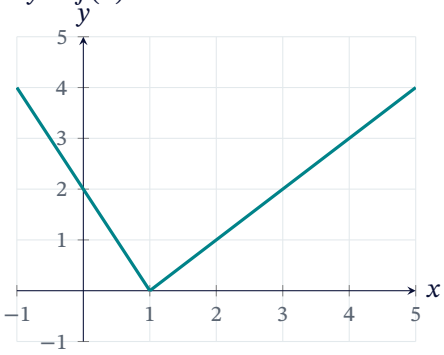
Example 02

Video · E02

The graph of f is made of two line segments. Let $g(x) = x^2$ and $h(x) = f(g(x))$.
(a) Find $h'(2)$.
(b) Find $h'(0)$.

> Find the inside value $g(a)$ first, then the slope of f there.

$y = f(x)$



Example 03

Video · E03

Using the table of Example 1, let $q(x) = x\,g(x^2)$. What is $q'(2)$?

> Product first, the chain rule inside.

A. 3
B. 4
C. 6
D. -6

VIDEO COMPLETE Check your example answers at the end of this booklet.

3.1b / Example answers

The Chain Rule with Tables and Graphs

E01 (a) 35; (b) 12; (c) $y = 2 + 35(x - 1)$

E02 (a) 4; (b) 0

E03 (C)