3.1a / Practice workbook

The Chain Rule

Practice

Problems 1–9 / Show your reasoning

1 Find the derivative of each function. (a) $y = (4x - 1)^5$ (b) $y = e^{x^2 - 3x}$ (c) $y = \sin(x^3)$

2 Let $f(x) = \sqrt{6x - 2}$. (a) Find $f'(3)$. (b) Write an equation of the line tangent to the graph of f at $x = 3$.

3 Find the slope of each graph at the given x. (a) $y = \log_3(2x + 1)$ at $x = 1$ (b) $y = \log_2(x^2 + 4)$ at $x = 2$

4 If $f(x) = \sqrt{1 + e^{2x}}$, then $f'(0) =$

A. $\frac{\sqrt{2}}{4}$
B. $\frac{\sqrt{2}}{2}$
C. $\sqrt{2}$
D. 2

5 If $k(x) = \ln(x^2 + 3x)$, then $k'(1) =$

A. $\frac{1}{5}$
B. $\frac{1}{4}$
C. $\frac{5}{4}$
D. 5

6 If $f(x) = 2^{3x}$, then $f'(1) =$

A. 12
B. 24
C. $8 \ln 2$
D. $24 \ln 2$

3.1a / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

The temperature of a cup of coffee is $T(t) = 20 + 60e^{-\frac{t}{10}}$ degrees Celsius, where t is measured in minutes.

(a) Find $T'(t)$.

(b) Find $T'(10)$. Using correct units, explain its meaning.

(c) At what time t is the coffee cooling at a rate of 3 degrees Celsius per minute?

3.1a / AP-style free response

Problem 8 ➤ */ Show the work and the reasons a grader looks for.*

A rider on a Ferris wheel is $h(t) = 30 - 25\cos\left(\frac{\pi t}{4}\right)$ feet above the ground, where t is measured in minutes.

(a) Find $h'(2)$. Using correct units, explain its meaning.

(b) At $t = 6$, is the rider rising or falling? Give a reason.

(c) For $0 < t < 8$, find the time at which $h'(t) = 0$.

9 Let $k(x) = (x-1)e^{2x}$. At which x-value does the graph of k have a horizontal tangent line?

A. 0
B. $\frac{1}{2}$
C. 1
D. 2

3.1a / Answer key

The Chain Rule

1 (a) $5(4x-1)^4 \cdot 4 = 20(4x-1)^4$; (b) $(2x-3)e^{x^2-3x}$; (c) $3x^2\cos(x^3)$

2 (a) $f'(3) = \frac{3}{4}$; (b) $y = 4 + \frac{3}{4}(x-3)$

3 (a) $\frac{2}{3\ln 3}$; (b) $\frac{1}{2\ln 2}$

4 (B)

5 (C)

6 (D)

7 (a) $T'(t) = 60e^{-\frac{t}{10}} \cdot \left(-\frac{1}{10}\right) = -6e^{-\frac{t}{10}}$; (b) $T'(10) = -\frac{6}{e}$ degrees Celsius per minute: at $t = 10$ minutes the temperature is decreasing at $\frac{6}{e} \approx 2.2$ degrees Celsius per minute; (c) $-6e^{-\frac{t}{10}} = -3$, $e^{-\frac{t}{10}} = \frac{1}{2}$, $t = 10\ln 2$ minutes

8 (a) $h'(2) = \frac{25\pi}{4}$ feet per minute: at $t = 2$ minutes the rider is rising at $\frac{25\pi}{4}$ feet per minute; (b) falling, because $h'(6) = -\frac{25\pi}{4} < 0$; (c) $t = 4$ minutes ($\sin\frac{\pi t}{4} = 0$)

9 (B)