3.1a / The Chain Rule

The chain rule with formulas · Part 1

Watch the lesson ➤

Learning goal

Differentiate a composite function with the chain rule, one layer at a time, and use it for slopes and tangent lines.

Key ideas

01 Is $(\sin 2x)'$ just $\cos 2x$? $\sin 2x$ runs the same wave **twice** as fast: at 0 the slopes are 1 and 2. The inside $2x$ moves 2 times as fast as x, so $(\sin 2x)' = 2\cos 2x$.

02 A composite $y = f(g(x))$: g is the inside, f the **outside**. E.g. $(3x+1)^2 = 9x^2 + 6x + 1$ has derivative $18x + 6 = 2(3x+1) \cdot 3$: outside′, times inside′.

03 The chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$, or $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$. In words: outside′(inside **kept**) × inside′.

04 The common shapes: $\frac{d}{dx} e^{g(x)} = e^{g(x)} g'(x)$, $\frac{d}{dx} \ln g(x) = \frac{g'(x)}{g(x)}$, $\frac{d}{dx} \sin g(x) = \cos g(x) \cdot g'(x)$, $\frac{d}{dx} \sqrt{g(x)} = \frac{g'(x)}{2\sqrt{g(x)}}$.

05 Any base $a > 0$: $a^x = e^{x \ln a}$, so $\frac{d}{dx} a^x = a^x \ln a$; and $\log_a x = \frac{\ln x}{\ln a}$, so $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$. With an inside u: times u'.

06 More layers, one link at a time: $y = e^{\sin(4x)}$: outside $e^{\sin(4x)}$, middle $\cos(4x)$, inside 4.

Class notes

Remember

composite function
A function of a function, $f(g(x))$: g is the inside, f the outside.

chain rule
$\big(f(g(x))\big)' = f'(g(x))\,g'(x)$: the outside's derivative at the kept inside, times the inside's derivative.

3.1a / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Times the inside's derivative

Code C1

✗ $\frac{d}{dx}\sqrt{x^2+5} = \frac{1}{2\sqrt{x^2+5}}$.

✓ Times the inside's derivative $2x$: $\frac{2x}{2\sqrt{x^2+5}} = \frac{x}{\sqrt{x^2+5}}$.

3.1a / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
The chain rule on the common shapes	Name the outside and the inside, then outside′(inside kept) × inside′.	E03 · Practice 1, 9
a^x and $\log_a x$ with an inside	$(a^g)' = a^g \ln a \cdot g'$, $(\log_a g)' = \dfrac{g'}{g \ln a}$.	Practice 3, 6
A value, a slope or a tangent line of a composite	The derivative first, then put in a. The tangent line: the point from $f(a)$, the slope from $f'(a)$.	E01, E02 · Practice 2, 4, 5
A chain rule rate in context, with units	Differentiate with the chain rule, put in the time, say the time, the quantity, the direction and the units.	Practice 7, 8

Example 01

Video · E01

Let $f(x) = \sqrt{x^2 + 5}$.
(a) Find $f'(x)$.
(b) Write an equation of the tangent line to the graph of f at $x = 2$.

Outside $\sqrt{\ }$, inside $x^2 + 5$.

Example 02

Video · E02

Let $h(x) = x^2e^{-3x}$. At which x-values does the graph of h have a horizontal tangent line?

Horizontal means $h'(x) = 0$.

A. 0 only
B. $\frac{2}{3}$ only
C. 0 and $\frac{2}{3}$
D. 0 and -2

Example 03

Video · E03

If $f(x) = \sin^2(3x)$, then $f'\left(\frac{\pi}{12}\right) =$

Three layers: $[\sin(3x)]^2$.

A. 1
B. $\frac{3}{2}$
C. 3
D. $3\sqrt{2}$

VIDEO COMPLETE Check your example answers at the end of this booklet.

3.1a / Example answers

The Chain Rule

E01 (a) $f'(x) = \dfrac{x}{\sqrt{x^2+5}}$; (b) $y = 3 + \frac{2}{3}(x-2)$

E02 (C)

E03 (C)