2.9–2.10 / Practice workbook

The Quotient Rule and the Other Trig Derivatives

Practice

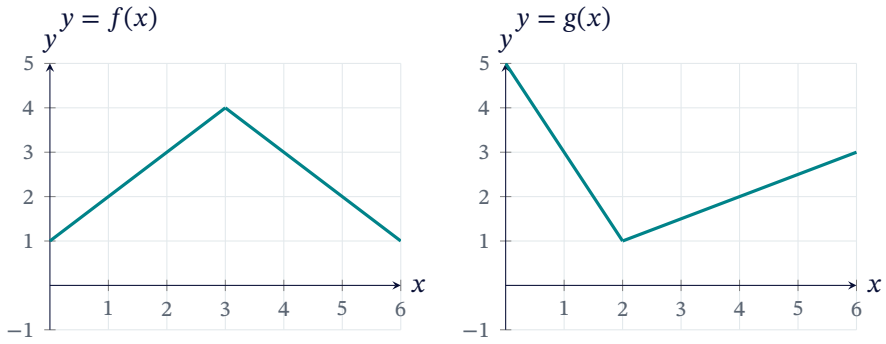
Problems 1–9 / Show your reasoning

1 Let $y = \dfrac{x}{x^2 + 4}$. (a) Find $\dfrac{dy}{dx}$. (b) At what positive value of x is the line tangent to the graph horizontal?

2 Let $g(x) = \dfrac{5x^3 - 2}{x^2}$. (a) Rewrite $g(x)$ without a fraction and find $g'(x)$. (b) Find $g'(-1)$. (c) Check your answer with the quotient rule.

3 Find the slope of the graph of $y = \sec x + \cot x$ at $x = \dfrac{\pi}{4}$.

4 The graphs of f and g are made of line segments. If $q(x) = \dfrac{f(x)}{g(x)}$, what is the value of $q'(4)$?



A. $\dfrac{7}{8}$
B. $-\dfrac{7}{8}$
C. $-\dfrac{7}{4}$
D. -2

5 If $f(x) = \csc x$, then $f'\left(\dfrac{\pi}{6}\right) =$

A. $-2\sqrt{3}$
B. $2\sqrt{3}$
C. -4
D. $\dfrac{2}{3}$

6 Which of the following is an equation of the line tangent to the graph of $y = \sec x$ at $x = \dfrac{\pi}{3}$?

A. $y = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$
B. $y = 2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$
C. $y = 2 + 4\left(x - \frac{\pi}{3}\right)$
D. $y = 2 - 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$

2.9–2.10 / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

Making x tables costs $C(x)$ dollars (values of C and C' in the table). The average cost per table is $A(x) = \dfrac{C(x)}{x}$.

x (tables)	40	50
$C(x)$ (dollars)	880	1000
$C'(x)$ (dollars per table)	10	12

(a) Find $A'(50)$. Show the work and units.

(b) Explain $A'(50)$ in context.

(c) Find $A(50)$. Why does $C'(50) < A(50)$ give $A'(50) < 0$?

2.9–2.10 / AP-style free response

Problem 8 ➤ / *Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{\sin x}{1 + \cos x}$ for $-\pi < x < \pi$.

(a) Show that $f'(x) = \dfrac{1}{1 + \cos x}$.

(b) Write an equation of the line tangent to the graph of f at $x = \dfrac{\pi}{2}$.

(c) Explain why every line tangent to the graph of f has a positive slope.

2.9–2.10 / AP-style free response

Problem 9 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{\cot x}{x}$. A calculator may be used.

(a) Your calculator has no cot key. Enter $\cot x$ as $\dfrac{1}{\tan x}$ and find $f'(1)$ to three decimal places.

(b) Check your answer to (a) with the quotient rule: write $f'(1)$ exactly.

2.9–2.10 / Answer key

The Quotient Rule and the Other Trig Derivatives

1 (a) $\frac{dy}{dx} = \frac{4-x^2}{(x^2+4)^2}$; (b) 2

2 (a) $g(x) = 5x - 2x^{-2}$, $g'(x) = 5 + 4x^{-3}$; (b) 1; (c) 1, the same as (b)

3 $\sqrt{2} - 2$

4 (B)

5 (A)

6 (B)

7 (a) -0.16 dollar per table, per table; (b) when 50 tables are made, the average cost per table is decreasing at a rate of 0.16 dollar per table, for each additional table; (c) $A(50) = 20$; $A'(50) = (C'(50) - A(50))/50 < 0$ because $12 < 20$

8 (a) shown: the top simplifies to $1 + \cos x$; (b) $y = 1 + (x - \pi/2)$; (c) $\cos x > -1$ on $(-\pi, \pi)$, so $1 + \cos x > 0$ and $f'(x) > 0$

9 (a) $f'(1) \approx -2.054$; (b) $f'(1) = \dfrac{(1)(-\csc^2 1) - (\cot 1)(1)}{1^2} = -\csc^2 1 - \cot 1$