Lecture Notes · Unit 2 · Lecture 9 AP CED 2.9–2.10

The Quotient Rule (and When to Skip It) + Derivatives of tan, cot, sec, csc | AP Calc AB 2.9-2

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

BUILD THE QUOTIENT RULE

A fraction's slope

$\frac{f}{g}$ → its slope → a new rule?

call it q: $q = \frac{f}{g} \to f = q \cdot g$ → a product

f, g, q differentiable, $g \neq 0$

$f' = (qg)' = q'g + qg'$

$q'g = f' - qg'$

$q'g^2 = gf' - (qg)g'$

$q'g^2 = gf' - fg'$

1. Quotient rule: $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}, g \neq 0$

$\left(\frac{1}{x}\right)' = \frac{x \cdot 0 - 1 \cdot 1}{x^2} = -\frac{1}{x^2}$

ORDER AND REWRITING

order, and when to skip it

2. order matters: gf' first

swap → the sign flips

square the bottom: g^2

3. a number or one power below → rewrite

$\frac{x^3 - 5}{4} = \frac{1}{4}x^3 - \frac{5}{4} \to \frac{3}{4}x^2$

$\frac{5}{\sqrt{x}} = 5x^{-\frac{1}{2}} \to -\frac{5}{2}x^{-\frac{3}{2}}$

a sum below → quotient rule

PROBLEM 1 OF 3

f, g are differentiable; values are in the table.

Let $q(x) = \frac{f(x)}{g(x)}$. Find $q'(2)$.

(A) $\frac{5}{3}$ (B) $-\frac{5}{3}$

(C) -5 (D) $-\frac{1}{2}$

x	2	5
$f(x)$	6	−1
$f'(x)$	−1	4
$g(x)$	3	2
$g'(x)$	2	−3

Solution

frame first → then $x = 2$

$q'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$

$= \frac{(3)(-1) - (6)(2)}{3^2}$

$= \frac{-3 - 12}{9} = \frac{-15}{9} = -\frac{5}{3}$

So (B)

Workbook
one like this

Common mistake

x	2	5
$f(x)$	6	−1
$f'(x)$	−1	4
$g(x)$	3	2
$g'(x)$	2	−3

✗ Common mistake

$$\frac{(6)(2)-(3)(-1)}{3^2}=\frac{5}{3}$$

✓ Do this

$$gf'-fg' \rightarrow \frac{(3)(-1)-(6)(2)}{3^2}=-\frac{5}{3}$$

Bottom $\rightarrow 3^2$

PROBLEM 2 OF 3

(a) Use $\tan x=\dfrac{\sin x}{\cos x}$ to show $(\tan x)'=\sec^2 x$.

(b) In the same way, find $(\cot x)'$.

(c) Write an equation of the tangent line to $y=\tan x$ at $x=\dfrac{\pi}{4}$.

$$\left(\frac{f}{g}\right)'=\frac{gf'-fg'}{g^2}$$

$(\sin x)'=\cos x,\ (\cos x)'=-\sin x$

$\sin^2 x+\cos^2 x=1$

Solution

$\dfrac{\sin x}{\cos x} \rightarrow$ rule $\rightarrow \sin^2+\cos^2=1$

(a)

$$(\tan x)'=\left(\frac{\sin x}{\cos x}\right)'$$

$$=\frac{\cos x\cdot\cos x-\sin x\cdot(-\sin x)}{\cos^2 x}$$

$$=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$

$$=\frac{1}{\cos^2 x}$$

$$=\sec^2 x,\ \cos x\neq 0$$

Your turn → workbook

Solution

$$\left(\frac{f}{g}\right)'=\frac{gf'-fg'}{g^2}$$

$(\sin x)'=\cos x,\ (\cos x)'=-\sin x$

$\sin^2 x+\cos^2 x=1$

(b)

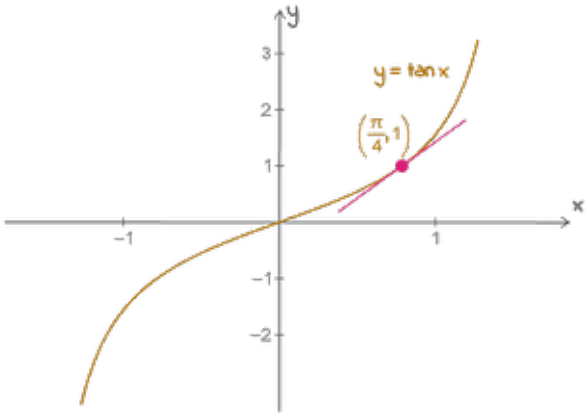
$$(\cot x)'=\frac{\sin x(-\sin x)-\cos x\cos x}{\sin^2 x}$$

$$=\frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x}=-\frac{1}{\sin^2 x}$$

$$=-\csc^2 x,\ \sin x\neq 0$$

Your turn
workbook

Solution



(c)

$$y=f(a)+f'(a)(x-a)$$

$$f\left(\frac{\pi}{4}\right)=\tan\frac{\pi}{4}=1$$

$$f'\left(\frac{\pi}{4}\right)=\sec^2\frac{\pi}{4}=\frac{1}{\cos^2\frac{\pi}{4}}$$

$$\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2} \rightarrow \left(\frac{\sqrt{2}}{2}\right)^2=\frac{2}{4}=\frac{1}{2}$$

$$f'\left(\frac{\pi}{4}\right) = \frac{1}{\frac{1}{2}} = 2$$

$$y = 1 + 2\left(x - \frac{\pi}{4}\right)$$

workbook → same type

ALL SIX TRIG DERIVATIVES

sec, csc, and all six

$$(\sec x)' = \frac{\cos x \cdot 0 - 1 \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \tan x$$

$$(\csc x)' = \frac{\sin x \cdot 0 - 1 \cdot \cos x}{\sin^2 x} = -\csc x \cot x$$

$(\sin x)' = \cos x \quad (\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x \quad (\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \tan x \quad (\csc x)' = -\csc x \cot x$

6 trig rules · co- → −

PROBLEM 3 OF 3

Let $f(x) = \dfrac{x-1}{x+1}$.

(a) Find $f'(x)$.

(b) Find all points on the graph of f where the tangent line is parallel to $y = 2x$.

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

Solution

sum below → quotient rule → $f' = 2$

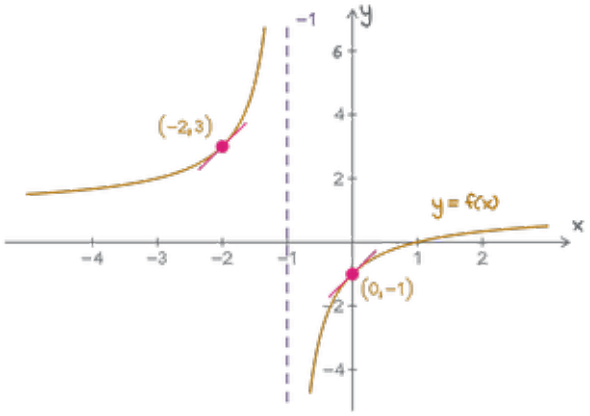
$x \neq -1$

(a)

$$f'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$= \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

Solution



(b)

$$\frac{2}{(x+1)^2} = 2$$

$$(x+1)^2 = 1 \rightarrow x+1 = 1 \text{ or } x+1 = -1$$

$$x = 0 \text{ or } x = -2$$

$$f(0) = \frac{-1}{1} = -1, \quad f(-2) = \frac{-3}{-1} = 3$$

$(0,-1), (-2,3)$

workbook
one like this

WRAP-UP

next: 3.1

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.