2.9–2.10 / The Quotient Rule and the Other Trig Derivatives

Watch the lesson ➤

The quotient rule; derivatives of tangent, cotangent, secant, and cosecant

Learning goal

Use the quotient rule with formulas, tables, and graphs, know when to rewrite instead, and find the derivatives of tan, cot, sec, and csc.

Key ideas

01 The **quotient rule**: $\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$, where $g(x) \neq 0$.

02 The **order** matters: bottom first, then minus top. Check with $\left(\frac{1}{x}\right)' = \frac{x\cdot 0 - 1\cdot 1}{x^2} = -\frac{1}{x^2}$, the same as the power rule.

03 When the bottom is a number or one power of x, **rewrite** first: $\frac{x^3 - 5}{4} = \frac{1}{4}x^3 - \frac{5}{4}$.

04 $\tan x = \frac{\sin x}{\cos x}$ gives $(\tan x)' = \sec^2 x$, and $\sec x = \frac{1}{\cos x}$ gives $(\sec x)' = \sec x \tan x$.

05 $(\cot x)' = -\csc^2 x$ and $(\csc x)' = -\csc x \cot x$: the **co**-functions get a minus.

06 With a table or a graph, write the **structure** first, then put in the numbers.

Remember

quotient rule
$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$: bottom times derivative of top, minus top times derivative of bottom, over bottom squared.

rewrite
Change the form before you differentiate, for example a fraction into powers of x.

co-function
cot, csc (and cos): their derivatives have a minus sign.

Two quick checks

$$\left(\frac{1}{x}\right)' = \frac{x\cdot 0 - 1\cdot 1}{x^2} = -\frac{1}{x^2} = \left(x^{-1}\right)'$$
$$\left(\frac{x^3-5}{4}\right)' = \left(\frac{1}{4}x^3 - \frac{5}{4}\right)' = \frac{3}{4}x^2$$
$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x$$

Class notes

2.9–2.10 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE

Write the structure, then the numbers: "$q'(2) = \frac{g(2)f'(2)-f(2)g'(2)}{[g(2)]^2} = \frac{(3)(-1)-(6)(2)}{3^2}$."

A tangent line or a derivative does not need to be simplified unless the question asks: $y = 1 + 2\left(x - \frac{\pi}{4}\right)$ is complete.

AVOID

Do not write $\left(\frac{f}{g}\right)' = \frac{f'}{g'}$.

Do not use the quotient rule when the bottom is a number: rewrite and use the constant multiple rule.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Bottom first, and square the bottom

Code D14

✗ $\left(\frac{x^2}{x+1}\right)' = \frac{x^2(1)-(x+1)(2x)}{x+1}$

✓ $\frac{(x+1)(2x)-x^2(1)}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$

02 Rewrite instead

Code D14

✗ $\left(\frac{x^4+1}{3}\right)' = \frac{3(4x^3)-(x^4+1)(0)}{9}$: right, but long, with more places to slip.

✓ $\frac{x^4+1}{3} = \frac{1}{3}x^4 + \frac{1}{3}$, so the derivative is $\frac{4}{3}x^3$.

03 Only the co-functions get a minus

Code D12

✗ $(\sec x)' = -\sec x \tan x$ and $(\cot x)' = \csc^2 x$.

✓ $(\sec x)' = \sec x \tan x$ and $(\cot x)' = -\csc^2 x$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 asks only for the quotient rule or for tan, cot, sec, or csc. The CED sample multiple-choice question 3 asks for a derivative rule from a table of f, g, f', and g' (the type of E01 and the first AP multiple-choice practice problem). The 2024 Chief Reader Report notes the quotient rule used where it was not needed.

2.9–2.10 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
$(f/g)'(a)$ from a table or from graphs	Write the structure $\frac{g(a)f'(a) - f(a)g'(a)}{[g(a)]^2}$ first: bottom first, then put in the numbers.	E01 · Practice 4
The quotient rule on a formula	Name top f and bottom g; $\frac{gf' - fg'}{g^2}$. Simplify the top only when you need to solve or read a sign.	Practice 1, 8
Rewrite first, instead of the quotient rule	Bottom a number or one power of x: split the fraction into powers of x, then the power rule.	E03 · Practice 2
Derivatives of $\tan x$, $\cot x$, $\sec x$, $\csc x$ and their values	Write $\tan x = \frac{\sin x}{\cos x}$, $\sec x = \frac{1}{\cos x}$, ... and use the quotient rule. The co-functions get a minus.	E02 · Practice 3, 5, 9
Tangent lines and rates with a quotient	The point uses the function value; the slope uses the quotient rule or a trig derivative.	Practice 6, 7

Example 01

Video · E01

f, g are differentiable; values are in the table. Let $q(x) = \frac{f(x)}{g(x)}$. Find $q'(2)$.

Bottom first. Then square the bottom.

x	2	5
$f(x)$	6	-1
$f'(x)$	-1	4
$g(x)$	3	2
$g'(x)$	2	-3

A. $\frac{5}{3}$

B. $-\frac{5}{3}$

C. -5

D. $-\frac{1}{2}$

Example 02

Video · E02

(a) Use $\tan x = \dfrac{\sin x}{\cos x}$ to show $(\tan x)' = \sec^2 x$.
(b) In the same way, find $(\cot x)'$.
(c) Write an equation of the tangent line to $y = \tan x$ at $x = \dfrac{\pi}{4}$.

Use $\cos^2 x + \sin^2 x = 1$ on the top.

Example 03

Video · E03

Let $f(x) = \dfrac{x-1}{x+1}$.
(a) Find $f'(x)$.
(b) Find all points on the graph of f where the tangent line is parallel to $y = 2x$.

Plan: bottom is a sum → quotient rule → $f'(x) = 2$

VIDEO COMPLETE Check your example answers at the end of this booklet.

2.9–2.10 / Example answers

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E01 (B)

E02 (a) shown: the top is $\cos^2 x + \sin^2 x = 1$, so $\sec^2 x$; (b) $-\csc^2 x$; (c) $y = 1 + 2(x - \pi/4)$

E03 (a) $2/(x+1)^2$; (b) $(0, -1), (-2, 3)$