2.8 / Practice workbook

The Product Rule

Practice

Problems 1–8 / Show your reasoning

1 Let $f(x) = x^3 e^x$. (a) Find $f'(x)$. (b) Find $f'(-1)$.

2 Write an equation of the line tangent to the graph of $y = \sqrt{x}\ \ln x$ at $x = 1$.

3 Selected values of a differentiable function f and its derivative are given in the table. If $h(x) = x^2 f(x)$, what is the value of $h'(3)$?

x	1	3
$f(x)$	5	2
$f'(x)$	2	-1

A. -6
B. -9
C. 3
D. 18

4 If $f(x) = x^2 \cos x$, then $f'\left(\frac{\pi}{2}\right) =$

A. $-\frac{\pi^2}{4}$
B. $-\pi$
C. $\frac{\pi^2}{4}$
D. 0

5 Let $p(x) = x\,g(x)$, where g is differentiable, $g(2) = -1$, and $g'(2) = 3$. Which of the following is an equation of the line tangent to the graph of p at $x = 2$?

A. $y = -2 + 3(x - 2)$
B. $y = -2 + 5(x - 2)$
C. $y = 5(x - 2)$
D. $y = -1 + 5(x - 2)$

2.8 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

At time t months, each lamp in a warehouse is worth $p(t)$ dollars and there are $q(t)$ lamps (table). The total value is $V(t) = p(t)\,q(t)$ dollars.

t (months)	0	2	5
$p(t)$	46	40	31
$p'(t)$	-3	-3	-2
$q(t)$	130	150	180
$q'(t)$	10	12	5

(a) Find $V'(2)$. Show the work and units.

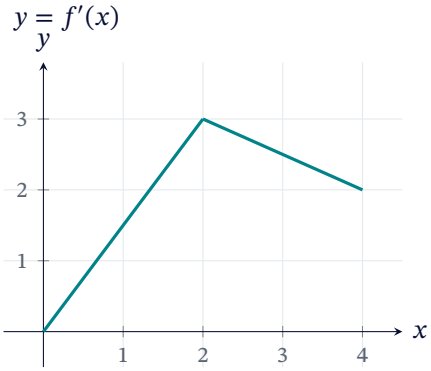
(b) Explain $V'(2)$ in context.

(c) Is the total value increasing or decreasing at $t = 5$? Why?

2.8 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

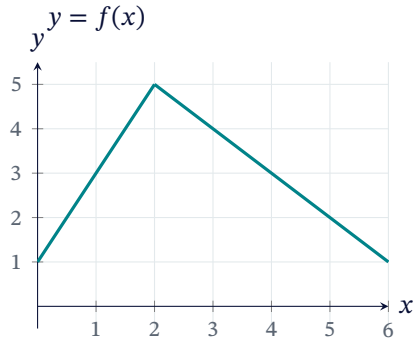
The graph of f', the derivative of f, is shown; it is made of two line segments. Also, $f(2) = 3$. Let $g(x) = x^2 f(x)$. Write an equation of the line tangent to the graph of g at $x = 2$.

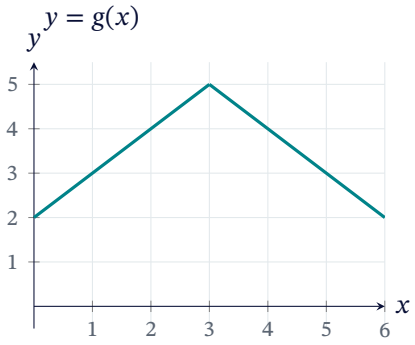


2.8 / AP-style free response

Problem 8 ➤ */ Show the work and the reasons a grader looks for.*

The graphs of f and g are made of line segments. Let $p(x) = f(x)\,g(x)$.





(a) Find $p'(1)$.

(b) Is $p'(5)$ positive, negative, or zero? Show the work that leads to your answer.

(c) Explain why the product rule cannot be used to find $p'(3)$.

2.8 / Answer key

The Product Rule

1 (a) $f'(x) = 3x^2e^x + x^3e^x = x^2e^x(x+3)$; (b) $\frac{2}{e}$

2 $y = 0 + 1(x-1)$, or $y = x - 1$

3 (C)

4 (A)

5 (B)

6 (a) $V'(2) = p'(2)q(2) + p(2)q'(2) = (-3)(150) + (40)(12) = 30$ dollars per month; (b) at $t = 2$ months, the total value of the lamps is increasing at a rate of 30 dollars per month; (c) decreasing, because $V'(5) = (-2)(180) + (31)(5) = -205 < 0$

7 $g(2) = 12$, $g'(2) = 2(2)(3) + (2)^2(3) = 24$: $y = 12 + 24(x-2)$

8 (a) $f'(1)g(1) + f(1)g'(1) = (2)(3) + (3)(1) = 9$; (b) $f'(5)g(5) + f(5)g'(5) = (-1)(3) + (2)(-1) = -5$: negative; (c) g has a corner at $x = 3$, so $g'(3)$ does not exist