Lecture Notes · Unit 2 · Lecture 8 AP CED 2.8

The Product Rule: Why It's Not f'·g' (Formulas, Tables, Graphs) | AP Calc AB 2.8

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TODAY'S QUESTION

Sum rule. Product rule?

$(f+g)' = f' + g'$

$(fg)' = f'g'$?

$(x \cdot x)' = (x^2)' = 2x$

$(x)' \cdot (x)' = (1)(1) = 1 \neq 2x$

$(5x)' = 5,\ 5' \cdot x' = 0 \cdot 1 = 0$

→ not $f'g'$. Then what?

FROM THE DEFINITION

why two terms

$$(fg)' = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

subtract & add $f(x)g(x+h)$ → adds 0

$f(x+h)g(x+h) - f(x)g(x+h)$

$+f(x)g(x+h) - f(x)g(x)$

$= [f(x+h) - f(x)]g(x+h) + f(x)[g(x+h) - g(x)]$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \cdot g(x+h) + f(x) \cdot \frac{g(x+h) - g(x)}{h}\right]$$

→ $f'(x)$ and $g'(x)$

$g(x+h) \to g(x)$

$(fg)' = f'g + fg'$

THE PRODUCT RULE

1. The product rule

$(fg)' = f'g + fg'$, f, g differentiable

first′ · second + first · second′

1st → only f moves; 2nd → only g moves

$(x \sin x)' = (x)' \sin x + x(\sin x)'$

$= (1)\sin x + x\cos x$

2. structure first

$(fg)'(a) = f'(a)g(a) + f(a)g'(a)$

PROBLEM 1 OF 2

f and g are differentiable; values are in the table. Let $h(x) = f(x)\,g(x) - 4g(x)$.

(a) Find $h'(3)$.

(b) Write an equation of the tangent line to the graph of h at $x = 3$.

x	1	3
$f(x)$	4	2
$f'(x)$	3	−1
$g(x)$	−2	5
$g'(x)$	1	4

Solution

structure → $x = 3$

(a)

$h'(x) = f'g + fg' - 4g'$

$h'(3) = (-1) \cdot 5 + 2 \cdot 4 - 4 \cdot 4$

$= -5 + 8 - 16 = -13$

(b)

$h(3) = 2 \cdot 5 - 4 \cdot 5 = -10$

$y = -10 - 13(x - 3)$

Workbook

one like this

Common mistake

x	1	3
f(x)	4	2
f'(x)	3	−1
g(x)	−2	5
g'(x)	1	4

X Common mistake

$f'(3)g'(3) - 4g'(3)$

$= -4 - 16 = -20$ X

✓ Do this

$f'g + fg' - 4g' \to -13$

AP guide → same slip

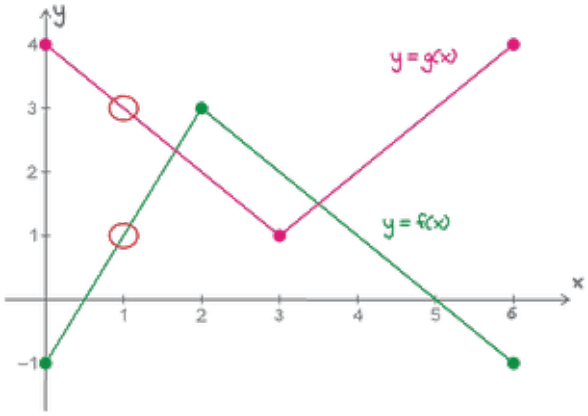
PROBLEM 2 OF 2

The graphs of f and g are made of line segments. Let $k(x) = f(x)\,g(x)$.

(a) Find $k'(1)$.

(b) Find $k'(5)$.

(c) Explain why the product rule cannot be used to find $k'(2)$.



Solution

height → value slope → derivative

(a)

$k'(1) = f'(1)g(1) + f(1)g'(1)$

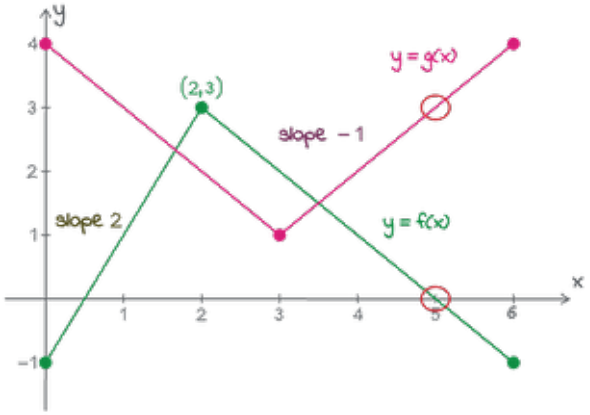
$f(1) = 1,\ g(1) = 3$

$f'(1) = 2,\ g'(1) = -1$

$= (2)(3) + (1)(-1) = 5$

Your turn → workbook

Solution



(b)

$k'(5) = f'(5)g(5) + f(5)g'(5)$

$f = 0 \quad g = 3 \quad f' = -1 \quad g' = 1$

$= (-1)(3) + 0 = -3$

(c)

f at x = 2 → corner

f'(2) undefined → no product rule

workbook

one like this

WRAP-UP

workbook · video → 2.9 quotient rule

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.