2.8 / The Product Rule

The product rule

Watch the lesson ➤

> **Learning goal**
> Use the product rule with formulas, tables, and graphs, and show its structure before the numbers.

Key ideas

01 The **product rule**: $\big(f(x)g(x)\big)' = f'(x)g(x) + f(x)g'(x)$. It has **two** terms.

02 Why two terms: in the difference quotient of $f(x)g(x)$, add and subtract $f(x)g(x+h)$. The two pieces go to $f'(x)g(x)$ and $f(x)g'(x)$, because $g(x+h) \to g(x)$: g is differentiable, so **continuous**.

03 The derivative of a product is **not** the product of the derivatives: $(x \cdot x)' = 2x$, but $1 \cdot 1 = 1$.

04 With a table or a graph, write the **structure** $f'(a)g(a) + f(a)g'(a)$ first, then put in the numbers.

05 A constant factor is a product too: $\big(4g(x)\big)' = 0 \cdot g(x) + 4g'(x) = 4g'(x)$.

Remember

product rule
$(uv)' = u'v + uv'$: derivative of the first times the second, plus the first times the derivative of the second.

factor
One of the functions that are multiplied.

structure
The formula written with f, g, f', g' before any numbers go in.

Why two terms

$$A = uv \;\to\; (u + \Delta u)(v + \Delta v)$$
$$\Delta A = u\,\Delta v + v\,\Delta u + \Delta u\,\Delta v$$
$$\frac{\Delta A}{\Delta x} = u\frac{\Delta v}{\Delta x} + v\frac{\Delta u}{\Delta x} + \Delta u\frac{\Delta v}{\Delta x}$$
$$\Delta x \to 0 \;:\; (uv)' = uv' + vu' + 0$$

Class notes

2.8 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the structure, then the numbers: "$h'(3) = f'(3)g(3) + f(3)g'(3) = (-1)(5) + (2)(4)$." The structure shows the method.	Do not write $(fg)' = f'g'$. The derivative of a product is not the product of the derivatives.
Use parentheses around each number you put in: $(-1)(5)$, not $-1 \cdot 5$ run together.	Do not write a number as a function: write $h'(3) = -13$, not $h'(x) = -13$.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Not the product of the derivatives

Code D13

✗ $f(2) = 3, f'(2) = -2, g(2) = 1, g'(2) = 5$, so $(fg)'(2) = f'(2)g'(2) = (-2)(5) = -10$.

✓ Two terms: $(fg)'(2) = f'(2)g(2) + f(2)g'(2) = (-2)(1) + (3)(5) = 13$.

02 A constant factor still counts

Code D10

✗ $h = fg - 4g$: "$-4g$ is a constant term," so $h' = f'g + fg'$.

✓ $-4g(x)$ changes with x: its derivative is $-4g'(x)$. $h' = f'g + fg' - 4g'$.

03 The point uses the product too

Code D5

✗ Tangent to $p(x) = xg(x)$ at $x = 3$ with $g(3) = 1, g'(3) = 2$: $p'(3) = 7$, so $y = 1 + 7(x - 3)$.

✓ The height is $p(3) = 3g(3) = 3$: $y = 3 + 7(x - 3)$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

The CED sample multiple-choice question 3 asks for the derivative of h = fg + 2g from a table, the type of E01 and the first AP multiple-choice practice problem. The CED sample FRQ 2(d) reads a product from graphs (the type of E02). Released FRQs use the product rule inside larger questions, such as 2023 AB5(b) and 2025 AB5.

2.8 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
$h'(a)$ from a table of f, g, f', g'	Write the structure $h'(a) = f'(a)g(a) + f(a)g'(a)$ first, then put in the four numbers.	E01 · Practice 3
$(fg)'(a)$ from the graphs of f and g	Values from the heights; $f'(a)$ and $g'(a)$ from the slopes of the segments through $x = a$. Not at a corner.	E02 · Practice 8
The product of two known functions (a power, sin, cos, e^x, $\ln x$)	Name the two factors u and v, find u' and v', then $u'v + uv'$.	Practice 1, 4
Tangent line or rate built on a product	The point uses $h(a) = f(a)g(a)$; the slope uses the product rule.	Practice 2, 5, 6, 7

Example 01

Video · E01

f and g are differentiable; values are in the table. Let $h(x) = f(x)\,g(x) - 4g(x)$.
(a) Find $h'(3)$.
(b) Write an equation of the tangent line to the graph of h at $x = 3$.

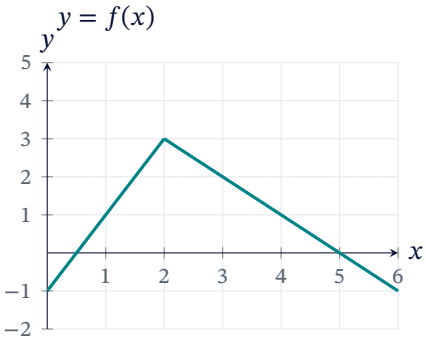
x	1	3
$f(x)$	4	2
$f'(x)$	3	-1
$g(x)$	-2	5
$g'(x)$	1	4

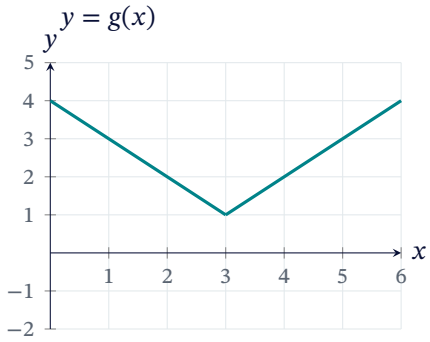
Write the structure first. Which column do you need?

Example 02

Video · E02

The graphs of f and g are made of line segments. Let $k(x) = f(x)\,g(x)$.
(a) Find $k'(1)$.
(b) Find $k'(5)$.
(c) Explain why the product rule cannot be used to find $k'(2)$.





> Heights give the values. Slopes give the derivatives.

VIDEO COMPLETE Check your example answers at the end of this booklet.

2.8 / Example answers

The Product Rule

E01 (a) -13; (b) $y = -10 - 13(x - 3)$

E02 (a) 5; (b) -3; (c) f has a corner at $x = 2$ (slopes 2 and -1), so $f'(2)$ does not exist and the rule needs $f'(2)$