2.7b / Practice workbook

A Limit That Is a Derivative

Practice

Problems 1–8 / Show your reasoning

1 Each limit is $F'(a)$ for some function F and number a. Name F and a, and evaluate the limit.

(a) $\lim_{x\to\pi} \dfrac{\sin x}{x-\pi}$

(b) $\lim_{x\to e} \dfrac{\ln x - 1}{x-e}$

2 Evaluate each limit.

(a) $\lim_{x\to\frac{\pi}{4}} \dfrac{\frac{\sqrt{2}}{2} - \sin x}{x-\frac{\pi}{4}}$

(b) $\lim_{x\to -1} \dfrac{x^5+1}{x+1}$

3 $\lim_{x\to\frac{\pi}{2}} \dfrac{\cos x}{x-\frac{\pi}{2}}$ is

A. 1
B. 0
C. -1
D. nonexistent

4 $\lim_{x\to 8} \dfrac{\sqrt[3]{x}-2}{x-8}$ is

A. 2
B. $\dfrac{1}{3}$
C. $\dfrac{1}{6}$
D. $\dfrac{1}{12}$

5 $\lim_{x\to 4} \dfrac{2-\sqrt{x}}{x-4}$ is

A. $\dfrac{1}{4}$
B. $-\dfrac{1}{4}$
C. 0
D. $-\dfrac{1}{2}$

2.7b / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

Let $g(x) = 2\sin x + x^2$.

(a) Find $g'(x)$.

(b) Find $\lim_{h\to 0} \dfrac{g(\pi + h) - g(\pi)}{h}$.

(c) Find $\lim_{x\to \frac{\pi}{2}} \dfrac{2\sin x + x^2 - 2 - \frac{\pi^2}{4}}{x - \frac{\pi}{2}}$. Show the work that leads to your answer.

2.7b / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

The function f is differentiable, and $f'(3) = 4$. Find each limit.

(a) $\lim_{h \to 0} \dfrac{f(3+2h) - f(3)}{h}$

(b) $\lim_{h \to 0} \dfrac{f(3) - f(3+5h)}{5h}$

(c) $\lim_{x \to 3} \dfrac{f(x) - f(3)}{x^2 - 9}$

8 Evaluate each limit by reading it as a derivative.

(a) $\lim_{h \to 0} \dfrac{\ln(e+h) - 1}{h}$

(b) $\lim_{h \to 0} \dfrac{\sqrt{9+h} - 3}{h}$

2.7b / Answer key

A Limit That Is a Derivative

1 (a) $F(x) = \sin x$, $a = \pi$, $F(\pi) = 0$: $\cos \pi = -1$; (b) $F(x) = \ln x - 1$, $a = e$, $F(e) = 0$: $\dfrac{1}{e}$

2 (a) $F(x) = \dfrac{\sqrt{2}}{2} - \sin x$: $F'\left(\dfrac{\pi}{4}\right) = -\cos \dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$; (b) $F(x) = x^5 + 1$, $a = -1$: $5(-1)^4 = 5$

3 (C)

4 (D)

5 (B)

6 (a) $g'(x) = 2\cos x + 2x$; (b) $g'(\pi) = 2\pi - 2$; (c) $g\left(\dfrac{\pi}{2}\right) = 2 + \dfrac{\pi^2}{4}$, so the limit is $g'\left(\dfrac{\pi}{2}\right) = \pi$

7 (a) $2f'(3) = 8$; (b) $-f'(3) = -4$; (c) $\dfrac{f'(3)}{3+3} = \dfrac{2}{3}$

8 (a) $f(x) = \ln x$, $a = e$: $f'(e) = \dfrac{1}{e}$; (b) $f(x) = \sqrt{x}$, $a = 9$: $f'(9) = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6}$