Lecture Notes · Unit 2 · Lecture 7 AP CED 2.7b

Limits That Are Secretly Derivatives: Read Them in One Line | AP Calc AB 2.7

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TODAY'S QUESTION

Zero over zero, nothing to factor

$$\lim_{x \to \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$$

plug in → $\frac{0}{0}$

factor? nothing to factor

WHOLE TOP = F

1. The whole top is F

$$\lim_{x \to 2} \frac{x^4 - 16}{x - 2}$$

top → 0, bottom → 0 → $\frac{0}{0}$, not done yet

whole top = one function → F

$F(x) = x^4 - 16$

$F(2) = 2^4 - 16 = 0$

$$\frac{x^4 - 16}{x - 2} = \frac{F(x) - F(2)}{x - 2}$$

bottom $\square - \triangle = x - 2$

$$\lim_{x \to 2} \frac{F(x) - F(2)}{x - 2} = F'(2)$$

$F'(x) = 4x^3 \rightarrow F'(2) = 4 \cdot 8 = 32$

32

x − a below · top = F F(a) = 0 → F'(a)

MINUS SIGNS

2. A minus sign? Still part of F

$$\lim_{x \to 2} \frac{16 - x^4}{x - 2}$$

$F(x) = 16 - x^4$

$F(2) = 16 - 16 = 0$

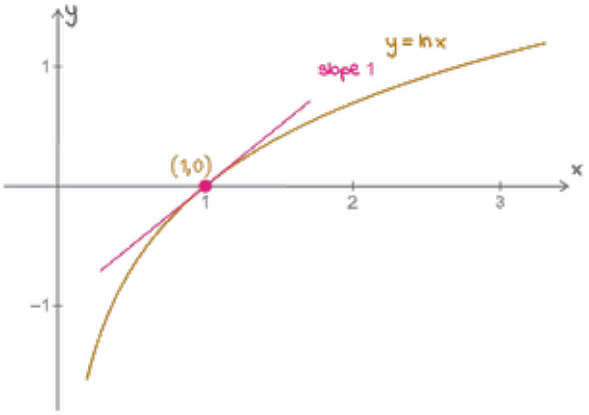
$$\lim_{x \to 2} \frac{F(x) - F(2)}{x - 2} = F'(2)$$

$F'(x) = 0 - 4x^3$

$F'(2) = -4 \cdot 8 = -32$

SLOPE, NOT HEIGHT

3. The limit is the slope, not the height



$$\lim_{x \to 1} \frac{\ln x}{x - 1}$$

ln 1 = 0 → limit 0? No.

whole top = F: F(x) = ln x, F(1) = ln 1 = 0

$$\lim_{x\to 1}\frac{F(x)-F(1)}{x-1}=F'(1)$$

$F'(x)=\frac{1}{x}\ \rightarrow\ F'(1)=1$

limit = 1, not 0

height $F(1)=0$

slope $F'(1)=1$ · not height

THE H FORM

4. The h form: square minus triangle

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=f'(a)$$

$\square-\triangle=(a+h)-a=h$

top → $f(a)-f(a)$ same input a → $f'(a)$

$$\lim_{h\to 0}\frac{f(a+3h)-f(a-h)}{4h}$$

$f'(a)$ exists

$(a+3h)-(a-h)=4h \Rightarrow \lim = f'(a)$

$$\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\frac{\pi}{6}}{h}$$

$f(x)=\sin x,\ a=\frac{\pi}{6}$

top → $\sin\frac{\pi}{6}-\sin\frac{\pi}{6}\rightarrow f'\left(\frac{\pi}{6}\right)$

$f'=\cos x,\quad \cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

limit = $\frac{\sqrt{3}}{2}$

PROBLEM 1 OF 2

Evaluate each limit by reading it as a derivative.

(a) $\lim_{x\to\frac{\pi}{3}}\dfrac{\cos x-\frac{1}{2}}{x-\frac{\pi}{3}}$

(b) $\lim_{h\to 0}\dfrac{\ln(2+h)-\ln 2}{h}$

whole top = F, $F(a)=0$

$\Rightarrow \lim_{x\to a}\frac{F(x)-F(a)}{x-a}=F'(a)$

$(\cos x)'=-\sin x$

Solution

top = F → $F(a)=0\ \rightarrow F'(a)$

(a)

$F(x)=\cos x-\frac{1}{2},\ a=\frac{\pi}{3}$

$F\left(\frac{\pi}{3}\right)\ \rightarrow\ \frac{1}{2}-\frac{1}{2}=0$

$$\lim_{x\to\frac{\pi}{3}}\frac{F(x)-F\left(\frac{\pi}{3}\right)}{x-\frac{\pi}{3}}=F'\left(\frac{\pi}{3}\right)$$

$F'(x)=-\sin x$

$F'\left(\frac{\pi}{3}\right)=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$

Your turn
workbook

Solution

$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=f'(a)$

$(\ln x)'=\frac{1}{x}$

(b)

$(2+h)-2=h$

top → $\ln 2-\ln 2$

$f(x)=\ln x,\ a=2\ \rightarrow\ f'(2)$

$$f'(x) = \frac{1}{x} \rightarrow f'(2) = \frac{1}{2}$$

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Common mistake: the top is flipped

✗ Common mistake

$$\lim_{h \to 0} \frac{\ln 2 - \ln(2+h)}{h} = \frac{1}{2} \; ?$$

✓ Do this

$$\text{top} = f(a) - f(a+h) \rightarrow a - (a+h) = -h$$

$$\rightarrow \; -f'(2) = -\frac{1}{2}$$

PROBLEM 2 OF 2

$\lim_{x \to 3} \frac{e^x - e^3}{x - 3}$ is

(A) 0 (B) $3e^2$

(C) e^3 (D) nonexistent

whole top = F, F(a) = 0

$\Rightarrow \lim_{x \to a} \frac{F(x) - F(a)}{x - a} = F'(a)$

$(e^x)' = e^x$

Solution

whole top = F

$$F(x) = e^x - e^3, \; a = 3, \; F(3) = 0$$

$$\lim_{x \to 3} \frac{F(x) - F(3)}{x - 3} = F'(3)$$

$$F'(x) = e^x - 0 = e^x$$

$$F'(3) = e^3 \rightarrow (C)$$

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WRAP-UP

workbook · video → 2.8 product rule

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.