2.7b / A Limit That Is a Derivative

Watch the lesson ➤

Interpreting a limit as the definition of a derivative · Part 2

Learning goal

Read a limit as the definition of a derivative of a function whose derivative you know, and evaluate it with a rule.

Key ideas

01 Substitution gives $\frac{0}{0}$? Call the **whole top** $F(x)$. The top goes to 0, so $F(a) = 0$.

02 Then $\dfrac{F(x)}{x-a} = \dfrac{F(x)-F(a)}{x-a}$, the average rate of change, and $\lim_{x\to a} \dfrac{F(x)}{x-a} = F'(a)$.

03 Use a rule you know. The limit is the **slope** $F'(a)$, not the value $F(a) = 0$.

04 A minus sign or a constant is part of the top: $F(x) = 16-x^4$ gives $F'(x) = -4x^3$. No rewriting.

05 Example: $\lim_{x\to 1} \dfrac{\ln x}{x-1}$ has $F(x) = \ln x$, $F(1) = 0$. The limit is $F'(1) = \dfrac{1}{1} = 1$.

06 AP also writes the h form $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$. By the definition, it is $f'(a)$.

Remember

definition of the derivative
The limit that gives $f'(a)$, in the h form or the x form.

recognize
See that an expression has a known shape, and use what you know about that shape.

indeterminate form
$\frac{0}{0}$ from direct substitution. It means "not done yet," not 0 and not "does not exist."

Class notes

2.7b / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE

Name the function and the number, then write the derivative: "With $F(x) = \sin x$, $F(\pi) = 0$, so the limit is $F'(\pi) = \cos \pi = -1$."

Keep lim until you replace the whole limit by $F'(a)$.

AVOID

Do not write a limit equal to $\frac{0}{0}$. Write "direct substitution gives 0/0," then recognize the derivative.

Do not write $F'(x) = -1$ when you mean the number $F'(\pi) = -1$.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The limit is the slope, not the value

Code D4

✗ $\lim_{x \to \frac{\pi}{2}} \dfrac{\cos x}{x - \frac{\pi}{2}} = \cos \dfrac{\pi}{2} = 0$

✓ With $F(x) = \cos x$, the limit is $F'\left(\dfrac{\pi}{2}\right) = -\sin \dfrac{\pi}{2} = -1$.

02 The minus sign is part of the top

Code D2

✗ $\lim_{x \to 1} \dfrac{e - e^x}{x - 1} = e$

✓ The whole top is $F(x) = e - e^x$, so $F'(x) = -e^x$ and the limit is $F'(1) = -e$.

03 0/0 does not mean "does not exist"

Code D4

✗ $\lim_{x \to 3} \dfrac{e^x - e^3}{x - 3}$: substitution gives $\frac{0}{0}$, so the limit does not exist.

✓ 0/0 means not done yet. With $F(x) = e^x - e^3$, $F(3) = 0$, the limit is $F'(3) = e^3$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 asks to evaluate a limit by reading it as a derivative. The 2012 Course Description AB multiple-choice question 1 asks which derivative a limit represents (the type of E02 and M01–M03).

2.7b / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Call the whole top F, then evaluate the limit with a rule	Substitution gives 0/0, so the whole top $F(x)$ has $F(a) = 0$. Then $\frac{F(x)}{x-a} = \frac{F(x)-F(a)}{x-a}$, and the limit is $F'(a)$.	E01 · Practice 1, 4, 8
A minus sign in front, or a bottom like $x + 1$ (still the whole top)	Do not rewrite. $F(x) = c - f(x)$ is still the whole top, and $F'(a) = -f'(a)$. A bottom $x + 1$ is $x - (-1)$, so $a = -1$.	Practice 2, 5, 7
The value is $F'(a)$, not $F(a) = 0$, and not "does not exist"	Direct substitution gives 0/0, which means "not done yet," not 0 and not "does not exist." The limit is the slope $F'(a)$.	E02 · Practice 3
Both forms from one known derivative (free response)	Find $g'(x)$ once. The h form $\frac{g(a+h)-g(a)}{h}$ is $g'(a)$ by the definition; for the x form, call the whole top F.	Practice 6

Example 01

Video · E01

Evaluate each limit by reading it as a derivative.

(a) $\lim_{x\to\frac{\pi}{3}} \dfrac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$

(b) $\lim_{h\to 0} \dfrac{\ln(2+h) - \ln 2}{h}$

> Plan: top $= F$, $F(a) = 0$, then $F'(a)$

Example 02

Video · E02

$\lim_{x \to 3} \dfrac{e^x - e^3}{x - 3}$ is

A. 0
B. $3e^2$
C. e^3
D. nonexistent

> $F(x) = e^x - e^3$. What is $F'(x)$?

VIDEO COMPLETE Check your example answers at the end of this booklet.

2.7b / Example answers

A Limit That Is a Derivative

E01 (a) -sqrt(3)/2; (b) $\frac{1}{2}$

E02 (C)