2.7a / Practice workbook

Derivatives of sin x, cos x, e^x, ln x, and a^x

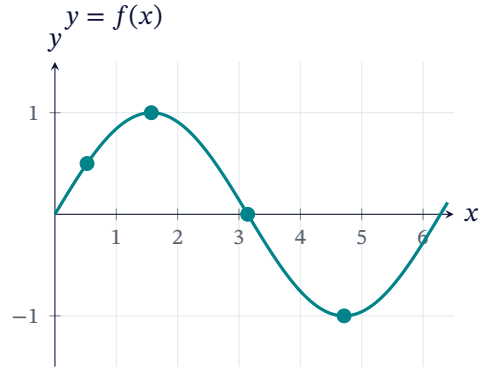
Practice

Problems 1–9 / Show your reasoning

1 Let $f(x) = x + 2\cos x$ for $0 \le x \le 2\pi$. (a) Find all values of x where the line tangent to the graph of f is horizontal. (b) Write an equation of the tangent line at the smaller of these values.

2 Let $f(x) = \sin x + \cos x$. (a) Find $f'(x)$. (b) Find all x with $0 \le x < 2\pi$ where the line tangent to the graph of f is horizontal.

3 The graph of $f(x) = \sin x$ is shown, with four points marked. At which of these values of x is the slope of the line tangent to the graph the greatest?



A. $\frac{\pi}{2}$
B. π
C. $\frac{\pi}{6}$
D. $\frac{3\pi}{2}$

4 The height of a buoy above the sea floor is $H(t) = 3 + 2\sin t$ meters, where t is the time in seconds. Which of the following is true at $t = \frac{\pi}{3}$?

A. The buoy is falling at 1 meter per second.
B. The buoy is rising at 1 meter per second.
C. The buoy is rising at $\sqrt{3}$ meters per second.
D. The buoy is rising at $3 + \sqrt{3}$ meters per second.

5 For which value of $a > 0$ does the line tangent to the graph of $y = \ln x$ at $x = a$ pass through the origin?

A. 1
B. $\frac{1}{e}$
C. e
D. There is no such value.

2.7a / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

The amount of a medicine in a patient's blood is $M(t) = 20 + 12\ln t - t$ milligrams, where t is the time in hours, $1 \le t \le 20$.

(a) Find $M'(4)$. Using correct units, explain the meaning of $M'(4)$ in the context of the problem.

(b) Is the amount of medicine increasing or decreasing at time $t = 15$? Give a reason for your answer.

(c) At what time t is the rate of change of the amount of medicine equal to 0?

2.7a / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

The function f is differentiable, and selected values of f' are given in the table. Let $k(x) = 3e^x - 2f(x)$.

x	0	1
$f'(x)$	5	-2

(a) Write $k'(x)$ in terms of $f'(x)$.

(b) Find $k'(0)$.

8 Let $f(x) = 5^x + x^5$.

(a) Find $f'(x)$.

(b) Find $f'(1)$.

(c) Write an equation of the line tangent to the graph of f at $x = 0$.

2.7a / AP-style free response

Problem 9 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = x + a\cos x$, where $a \neq 0$ is a constant.

(a) Show that the line $y = x + a$ is tangent to the graph of f at $x = 0$.

(b) Find all values of x in $0 \leq x \leq 4\pi$ where the line tangent to the graph of f is parallel to $y = x + a$. At which of these values is the line $y = x + a$ itself the tangent line?

2.7a / Answer key

Derivatives of sin x, cos x, e^x, ln x, and a^x

1 (a) $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$; (b) $y = \frac{\pi}{6} + \sqrt{3}$

2 (a) $f'(x) = \cos x - \sin x$; (b) $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$, where $\cos x = \sin x$

3 (C)

4 (B)

5 (C)

6 (a) $M'(4) = 2$: at $t = 4$ hours, the amount of medicine is increasing at a rate of 2 milligrams per hour; (b) decreasing, because $M'(15) = -\frac{1}{5} < 0$; (c) $t = 12$ hours

7 (a) $k'(x) = 3e^x - 2f'(x)$; (b) $k'(0) = 3 - 10 = -7$

8 (a) $f'(x) = 5^x \ln 5 + 5x^4$; (b) $f'(1) = 5 \ln 5 + 5$; (c) $f(0) = 1$, $f'(0) = \ln 5$: $y = 1 + (\ln 5)x$

9 (a) $f(0) = a$ (on the line) and $f'(0) = 1 - a \sin 0 = 1$ (same slope); (b) parallel at $x = 0, \pi, 2\pi, 3\pi, 4\pi$; the tangent line is $y = x + a$ at $x = 0, 2\pi, 4\pi$ (at $\pi, 3\pi$ it is $y = x - a$)