Lecture Notes · Unit 2 · Lecture 6 AP CED 2.7a

Derivatives of sin x, cos x, e^x, ln x (and a^x): Where They Come From | AP Calc AB 2.7

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

TODAY'S QUESTION

when the power rule stops

Power rule → only x^n

$\sin x$, $\cos x$, e^x, $\ln x$ → not powers → slope?

a^x → definition

SINE

$$\sin(x+h) = \sin x\cos h + \cos x\sin h$$

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

→ always radians

$$(\sin x)' = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1) + \cos x\sin h}{h}$$

$$= \lim_{h\to 0} \left[\sin x \cdot \frac{\cos h - 1}{h} + \cos x \cdot \frac{\sin h}{h}\right]$$

$$= \sin x \cdot 0 + \cos x \cdot 1 = \cos x$$

1. $(\sin x)' = \cos x$

COSINE

$$\cos(x+h) = \cos x\cos h - \sin x\sin h$$

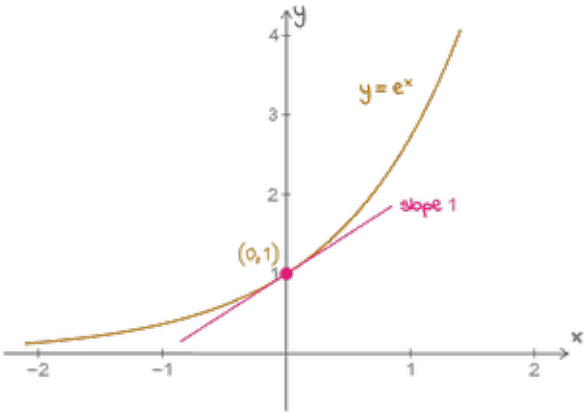
$$(\cos x)' = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} \left[\cos x \cdot \frac{\cos h - 1}{h} - \sin x \cdot \frac{\sin h}{h}\right]$$

$$= \cos x \cdot 0 - \sin x \cdot 1 = -\sin x$$

2. $(\cos x)' = -\sin x$ → minus

E TO THE X



$$\frac{e^{x+h} - e^x}{h}$$

$$= e^x \cdot \frac{e^h - 1}{h}$$

$\lim_{h\to 0} \frac{e^h - 1}{h}$ = slope of e^x at 0

e → tangent at 0 has slope 1

$$(e^x)' = e^x \cdot 1 = e^x$$

3. $(e^x)' = e^x$ → slope = height

x^e → base x → $e x^{e-1}$

e^x (x on top) → e^x

A^X AND LN X

a to the x, and ln x

a^x, $a > 0$

$(a^x)' = a^x \ln a$

$(2^x)' = 2^x \ln 2$

$a = e \rightarrow \ln e = 1 \rightarrow (e^x)' = e^x$

4. $(\ln x)' = \frac{1}{x}$, $x > 0$

$x = -2 \rightarrow \frac{1}{x} = -\frac{1}{2}$

$\ln(-2)$ undefined → no slope

COMBINING DERIVATIVE RULES

Sums and constant multiples

sums, multiples → same rules

$(3e^x - 2\cos x)' = 3e^x - 2(-\sin x)$

$= 3e^x + 2\sin x$

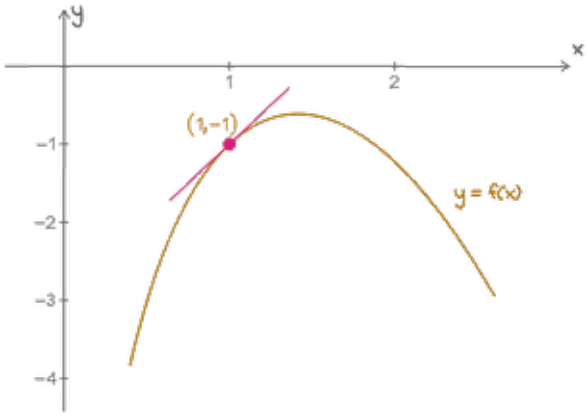
PROBLEM 1 OF 2

Let $f(x) = 4\ln x - x^2$ for $x > 0$.

(a) Find $f'(x)$.

(b) Write an equation of the tangent line at $x = 1$.

(c) Where is the tangent line horizontal?



Solution

terms → point, slope → $f' = 0$ → domain

(a)

$f'(x) = 4 \cdot \frac{1}{x} - 2x$

$= \frac{4}{x} - 2x$

(b)

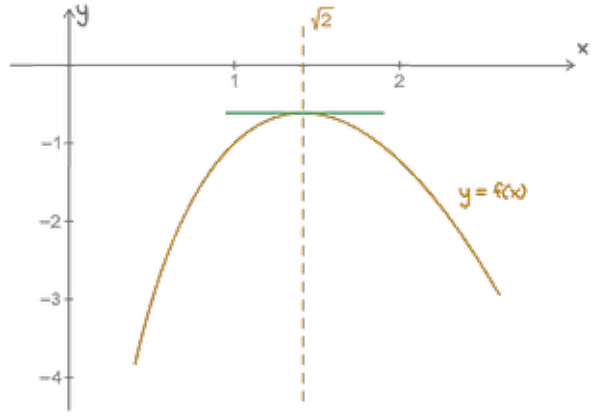
$f(1) = 4\ln 1 - 1^2$

$= 4(0) - 1 = -1$

$f'(1) = \frac{4}{1} - 2(1) = 2$

$y = f(a) + f'(a)(x - a) = -1 + 2(x - 1)$

Solution



(c)

$f'(x) = 0 \rightarrow \frac{4}{x} - 2x = 0$

$4 - 2x^2 = 0 \rightarrow x^2 = 2$

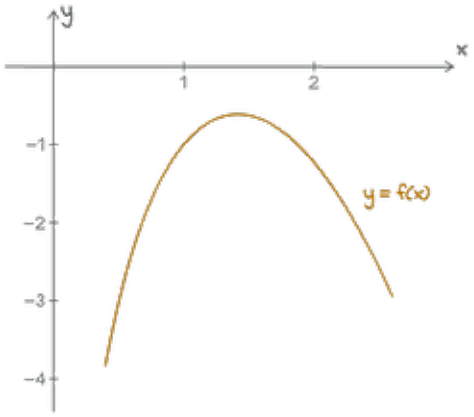
$x = \pm\sqrt{2}$

$\ln x$ needs $x > 0$

$x = \sqrt{2}$

Workbook
one like this

(c) Common mistake



X Common mistake

$x = \pm\sqrt{2} \rightarrow$ two flat spots?

✓ Do this

$x > 0$ only $\rightarrow x = \sqrt{2}$

$-\sqrt{2} \rightarrow$ outside the domain

PROBLEM 2 OF 2

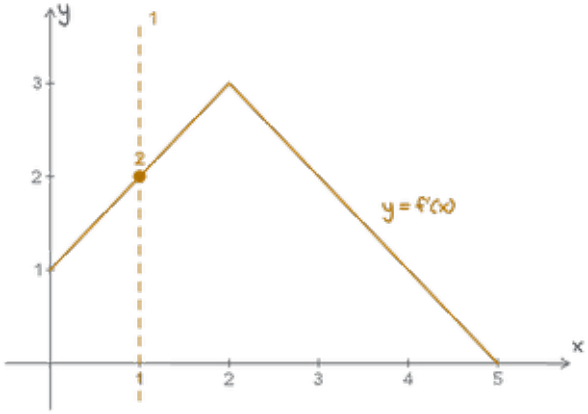
The graph of f', the derivative of f, is shown for $0 < x < 5$.

Let $g(x) = f(x) - 4\ln x$.

(a) Write $g'(x)$ in terms of $f'(x)$.

(b) Find $g'(1)$.

(c) Is the tangent line to the graph of g horizontal at $x = 4$? Justify.



Solution

formula → graph f′

(a)

$g'(x) = f'(x) - 4 \cdot \frac{1}{x} = f'(x) - \frac{4}{x}$

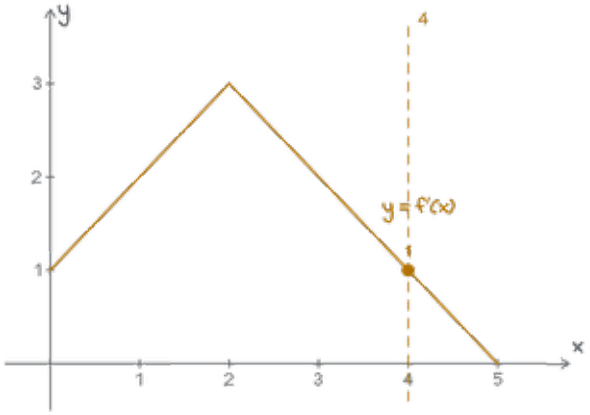
(b)

$g'(1) = f'(1) - \frac{4}{1}$

$f'(1) =$ height at $1 = 2$

$g'(1) = 2 - 4 = -2$

Solution



(c)

$g'(4) = 0?$ $g'(4) = f'(4) - \frac{4}{4}$

$f'(4) = 1$

$g'(4) = 1 - 1 = 0$

$g'(4) = 0 \rightarrow$ horizontal tangent to g at $x = 4$

Workbook
one like this

WRAP-UP

a^x · sums · tangents → 2.7 limit → derivative

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.