2.7a / Derivatives of sin x, cos x, e^x, ln x, and a^x

Watch the lesson ➤

Derivatives of sin x, cos x, e^x, ln x, and a^x · Part 1

Learning goal

Find derivatives that use $\sin x$, $\cos x$, e^x, $\ln x$, and a^x, know why the sine, cosine, and e^x rules hold, and use them for tangent lines and rates with units.

Key ideas

01 $\frac{d}{dx}\sin x = \cos x$. Why: with $\sin(x+h) = \sin x\cos h + \cos x\sin h$ and the limits $\frac{\sin h}{h} \to 1$, $\frac{\cos h - 1}{h} \to 0$, the **difference quotient** goes to $\cos x$.

02 $\frac{d}{dx}\cos x = -\sin x$. Note the **minus**: just right of $x = 0$, $\cos x$ goes down while $\sin x > 0$.

03 $\frac{d}{dx}e^x = e^x$: the slope equals the **height**. The number e is the base whose graph has slope 1 at $x = 0$.

04 $\frac{d}{dx}\ln x = \frac{1}{x}$, for $x > 0$ only: $\ln x$ is **defined** only for $x > 0$.

05 For $a > 0$: $\frac{d}{dx}a^x = a^x \ln a$ (why: Unit 3). With $a = e$, $\ln e = 1$ gives back e^x. In x^5 the **base** is x (power rule); in 5^x the exponent is x.

06 e^x is not a power of x, so the power rule does **not** apply. Sums and constant multiples work as before: $\frac{d}{dx}[3e^x - 2\cos x] = 3e^x + 2\sin x$.

Remember

e
The number (about 2.718) that makes the slope of $y = e^x$ at $x = 0$ equal to 1.

natural logarithm
$\ln x$: the power of e that gives x. Defined only for $x > 0$.

slope of a graph
The slope of the tangent line, $f'(x)$, at that point.

Class notes

2.7a / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the derivative before you substitute: $f'(x) = \frac{4}{x} - 2x$, so $f'(1) = 2$.	Do not write $f'(x) = 2$ when you mean the number $f'(1) = 2$.
Use parentheses when you substitute: $-2\sin\left(\frac{\pi}{3}\right)$, $4\ln(1) - (1)^2$.	Do not write an equals sign between things that are not equal, such as $f'(x) = -2\sin x = -\sqrt{3}$.
Give a rate with units and the time: "at $t = 4$ hours, increasing at 2 milligrams per hour."	

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The minus sign of cosine

Code D12

✗ $\frac{d}{dx}[2\cos x] = 2\sin x$

✓ $\frac{d}{dx}[2\cos x] = -2\sin x$. The "co" function gets the minus.

02 e^x is not a power

Code D11

✗ $\frac{d}{dx}e^x = xe^{x-1}$

✓ The variable is in the exponent, so the power rule does not apply: $\frac{d}{dx}e^x = e^x$.

03 Stay in the domain of $\ln x$

Code D11

✗ $f(x) = 4\ln x - x^2$: $f'(x) = 0$ at $x = \sqrt{2}$ and $x = -\sqrt{2}$.

✓ $\ln x$ needs $x > 0$, so only $x = \sqrt{2}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 tests these four rules alone; they appear inside larger FRQs, such as 2025 AB2(d) and 2026 AB4(a). The 2012 BC Practice Exam multiple-choice question 77 asks for a derivative from basic rules (the type of E02).

2.7a / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Derivative of a sum built from $\sin x$, $\cos x$, e^x, $\ln x$, and powers	Term by term: $\sin \to \cos$, $\cos \to -\sin$, $e^x \to e^x$, $\ln x \to \frac{1}{x}$, $x^n \to nx^{n-1}$; constants in front stay.	E02 · Practice 7, 8
Tangent lines and slopes (horizontal tangent, the greatest slope, a tangent through a given point)	Slope $f'(a)$, point $(a, f(a))$. Horizontal tangent: solve $f'(x) = 0$ in the domain.	E01 · Practice 1, 3, 5, 9
A rate in context with units, and its sign	Find the derivative, put in the time, give units of the quantity per unit of time; a negative rate means decreasing at that instant.	Practice 4, 6
Why a rule holds (from the definition)	Write the difference quotient, use the angle-sum identity, group the terms, and use $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$.	Practice 2

Example 01

Video · E01

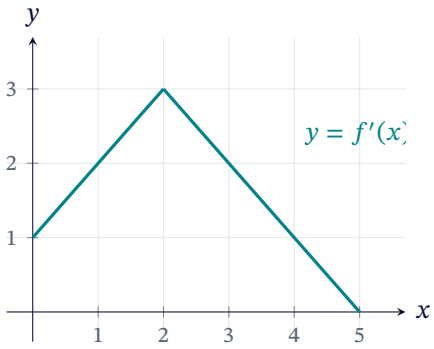
Let $f(x) = 4\ln x - x^2$ for $x > 0$.
(a) Find $f'(x)$.
(b) Write an equation of the tangent line at $x = 1$.
(c) Where is the tangent line horizontal?

> $\ln x$ lives only where $x > 0$.

Example 02

Video · E02

The graph of f', the derivative of f, is shown for $0 < x < 5$.
Let $g(x) = f(x) - 4\ln x$.
(a) Write $g'(x)$ in terms of $f'(x)$.
(b) Find $g'(1)$.
(c) Is the tangent line to the graph of g horizontal at $x = 4$? Justify.



> Plan: structure first → then read f' off the graph

VIDEO COMPLETE Check your example answers at the end of this booklet.

2.7a / Example answers

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E01 (a) $f'(x) = \dfrac{4}{x} - 2x$; (b) $y = -1 + 2(x - 1)$; (c) $x = \sqrt{2}$ (not $-\sqrt{2}$: $\ln x$ needs $x > 0$)

E02 (a) $f'(x) - 4/x$; (b) -2; (c) yes