2.5–2.6 / Practice workbook

The Power Rule and the Sum Rules

Practice

Problems 1–10 / Show your reasoning

1 Let $g(x) = 5\sqrt[3]{x^2} - \frac{8}{x} + \pi^2$. (a) Find $g'(x)$. (b) Find $g'(8)$.

2 Let $f(x) = x^4 - 4x^3 + 10$. (a) Find $f'(x)$ and write it in factored form. (b) Find every point on the graph of f where the tangent line is horizontal.

3 The line $y = 8x - 7$ is tangent to the graph of $f(x) = x^2 + bx + c$ at $x = 2$. Find the constants b and c.

4 Let $f(x) = ax^3 + 4x$, where a is a constant. The line tangent to the graph of f at $x = 1$ is parallel to the line $y = 10x + 3$. What is the value of a?

A. 9
B. 6
C. 2
D. $\frac{10}{3}$

5 Let $f(x) = \begin{cases} x^3 - x^2 + 2x, & x \leq 0 \\ 2x, & 0 < x \leq 3 \\ 2x + 1, & x > 3 \end{cases}$ At which values of x is f not differentiable?

A. $x = 3$ only
B. $x = 0$ only
C. $x = 0$ and $x = 3$
D. None; f is differentiable everywhere.

6 The volume of a spherical balloon is $V(r) = \frac{4}{3}\pi r^3$ cubic centimeters, where r is the radius in centimeters. What is the instantaneous rate of change of V with respect to r when $r = 6$ centimeters?

A. 288π cubic centimeters per centimeter
B. 144π cubic centimeters per centimeter
C. 48π cubic centimeters per centimeter
D. 144 cubic centimeters per centimeter

2.5–2.6 / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

The position of an object moving along a straight track is $s(t) = t^3 - 12t^2 + 36t + 1$ meters, where t is the time in seconds, $0 \le t \le 8$.

(a) Find $s'(1)$. Using correct units, explain the meaning of $s'(1)$ in the context of the problem.

(b) Find the two times t at which the rate of change of s is 0.

(c) Find the average rate of change of s over the interval between the two times in part (b). Indicate units of measure.

(d) The rate of change of s is 0 at both ends of that interval. Explain why the average rate of change over the interval is not 0.

2.5–2.6 / AP-style free response

Problem 8 ➤ / *Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} x^2 + ax, & x < 1 \\ bx^3 + 2, & x \geq 1 \end{cases}$ where a and b are constants, and f is differentiable at $x = 1$.

(a) Explain why $\lim_{x \to 1^-} f(x)$ must equal $f(1)$.

(b) Find the values of a and b. Show the work that leads to your answers.

(c) Write an equation of the line tangent to the graph of f at $x = 1$.

2.5–2.6 / AP-style free response

Problem 9 ➤ / *Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{x^2 - 4\sqrt{x}}{x}$ for $x > 0$.

(a) Rewrite $f(x)$ using powers of x, and find $f'(x)$.

(b) Write an equation of the line tangent to the graph of f at $x = 4$.

(c) Find the points where this tangent line crosses the x-axis and the y-axis.

2.5–2.6 / AP-style free response

Problem 10 ➤ / *Show the work and the reasons a grader looks for.*

The depth of water in a tank is $D(t) = \begin{cases} \frac{t^2}{4} + 2, & 0 \le t \le 4 \\ 3t - 6, & 4 < t \le 8 \end{cases}$ centimeters, where t is the time in hours.

(a) Find the average rate of change of D over the interval $0 \le t \le 8$. Indicate units of measure.

(b) Find $D'(2)$. Using correct units, explain its meaning in the context of the problem.

(c) Is D continuous at $t = 4$? Is D differentiable at $t = 4$? Justify your answers.

(d) Find $D'(6)$. Indicate units of measure.

2.5–2.6 / Answer key

The Power Rule and the Sum Rules

1 (a) $g'(x) = \frac{10}{3}x^{-1/3} + 8x^{-2}$; (b) $\frac{43}{24}$

2 (a) $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$; (b) $(0, 10)$ and $(3, -17)$

3 (a) $b = 4$; (b) $c = -3$

4 (C)

5 (A)

6 (B)

7 (a) $s'(1) = 15$: at $t = 1$ second, the position is increasing at 15 meters per second; (b) $t = 2$ and $t = 6$ seconds; (c) -8 meters per second; (d) the average rate of change is set by the change in position, $s(6) - s(2) = 1 - 33 = -32$; s' is 0 only at the two instants $t = 2$ and $t = 6$, not in between (for example $s'(4) = -12$)

8 (a) f is differentiable at $x = 1$, so f is continuous at $x = 1$; then $\lim_{x \to 1} f(x) = f(1)$, and so the left-hand limit equals $f(1)$; (b) $a = \frac{5}{2}$, $b = \frac{3}{2}$; (c) $y = \frac{7}{2} + \frac{9}{2}(x-1)$

9 (a) $f(x) = x - 4x^{-1/2}$, $f'(x) = 1 + 2x^{-3/2}$; (b) $f(4) = 2$, $f'(4) = \frac{5}{4}$: $y = 2 + \frac{5}{4}(x-4)$; (c) x-axis at $\left(\frac{12}{5}, 0\right)$; y-axis at $(0, -3)$

10 (a) $\frac{D(8) - D(0)}{8 - 0} = \frac{18 - 2}{8} = 2$ centimeters per hour; (b) $D'(2) = 1$: at $t = 2$ hours, the depth is increasing at 1 centimeter per hour; (c) continuous ($6 = 6$); not differentiable (slope 2 from the left, 3 from the right); (d) $D'(6) = 3$ centimeters per hour