2.5–2.6 / The Power Rule and the Sum Rules

Watch the lesson ➤

Applying the power rule; constant, sum, difference, and constant multiple rules

Learning goal

Differentiate powers, roots, and polynomials with the power, constant, sum, difference, and constant multiple rules, and use them for tangent lines, horizontal tangents, and constants at a joint.

Key ideas

01 The **power rule**: $\frac{d}{dx}(x^r) = rx^{r-1}$ for every real number r. Bring the exponent down, then subtract 1.

02 **Rewrite** first: $\sqrt[3]{x^2} = x^{2/3}$, $\frac{1}{x^4} = x^{-4}$, and divide each term by a single power of x.

03 A **constant** has derivative 0. A constant multiple stays in front: $\frac{d}{dx}(cf(x)) = cf'(x)$.

04 Sums and differences go **term** by term: $(f \pm g)' = f' \pm g'$. So every polynomial can be differentiated.

05 A **horizontal** tangent line is at an x where $f'(x) = 0$.

06 At a joint of a piecewise function made of polynomials, differentiable means the heights match (continuity) **and** the one-sided slopes match.

Remember

power rule
$\frac{d}{dx}x^r = rx^{r-1}$.

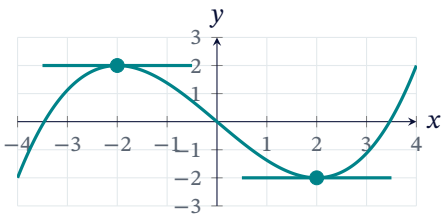
constant multiple
A number times a function, such as $5x^3$. The number stays when you differentiate.

horizontal tangent
A tangent line with slope 0: where $f'(x) = 0$.

joint
The x where a piecewise function changes from one formula to another.

Horizontal tangents are where $f'(x) = 0$

$(x+h)^2 = x^2 + 2xh + h^2$
$\frac{(x+h)^2 - x^2}{h} = 2x + h \to 2x$
$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$
$\frac{(x+h)^3 - x^3}{h} = 3x^2 + 3xh + h^2 \to 3x^2$



For $f(x) = \frac{1}{8}x^3 - \frac{3}{2}x$, $f'(x) = \frac{3}{8}x^2 - \frac{3}{2} = 0$ at $x = -2$ and $x = 2$.
The tangent lines there are horizontal: $y = 2$ and $y = -2$.

Class notes

2.5–2.6 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the derivative at a point as a number at that point: $f'(4) = \frac{33}{4}$, not $f'(x) = \frac{33}{4}$.	Do not write "f is continuous" at a joint without the reason; differentiability is the reason.
Write a tangent line from the point and the slope. You do not need to simplify: $y = 13 - 7(x - 2)$.	Do not simplify a tangent line equation you do not need to; simplifying is where sign errors happen.
Give the reason before you use continuity: "f is differentiable at $x = 1$, so f is continuous at $x = 1$."	

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Rewrite before the power rule

Code D9

✗ $\frac{d}{dx}\left(\frac{5}{x^2}\right) = \frac{5}{2x}$

✓ $\frac{5}{x^2} = 5x^{-2}$, so the derivative is $-10x^{-3} = -\frac{10}{x^3}$.

02 A constant alone gives 0

Code D10

✗ $\frac{d}{dx}\left(\pi^2\right) = 2\pi$

✓ π^2 is a number, so its derivative is 0.

03 Match the height and the slope

Code D8

✗ At $x = 3$ both pieces $2x$ and $2x + 1$ have slope 2, so f is differentiable at $x = 3$.

✓ The heights are 6 and 7: f is not continuous at $x = 3$, so it is not differentiable there.

Real AP questions on this topic Free on AP Central. Try them after the practice.

Basic derivatives with these rules appear inside larger released FRQs, such as 2023 AB5(d) and 2025 AB2(D), and tangent lines inside 2024 AB5(a). The CED sample multiple-choice question 2 asks where a piecewise function is not differentiable (the type of E03 and the second AP multiple-choice practice problem).

2.5–2.6 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Rewrite roots and quotients as powers, then use the power rule	Write every term as cx^r ($\sqrt[3]{x^2} = x^{2/3}$, $\frac{1}{x^4} = x^{-4}$, divide each term by a single power). Then bring r down and subtract 1.	E01 · Practice 1, 9
Polynomials term by term, with constants and constant multiples	Differentiate each term. A constant term gives 0; a constant in front stays in front.	
Tangent lines and horizontal tangents with the rules	Tangent: $y = f(a) + f'(a)(x - a)$. Horizontal tangent: solve $f'(x) = 0$, then find the heights $f(x)$.	E02 · Practice 2, 3, 4
Two constants that make a piecewise function differentiable	(i) Heights match at the joint (differentiable means continuous). (ii) One-sided slopes match. Solve the two equations.	E03 · Practice 5, 8
Sum and constant multiple rules on named functions (table values)	$(3f - 2g)'(a) = 3f'(a) - 2g'(a)$: use the derivative columns, not the value columns.	
A rate of change in context with the rules, with units	Differentiate with the rules, evaluate at the given input, and give units of the output per unit of the input.	Practice 6, 7, 10

Example 01

Video · E01

Let $f(x) = \dfrac{x^3 - 4\sqrt{x}}{x}$. What is the value of $f'(4)$?

A. 47

B. 7

C. 12

D. $\dfrac{33}{4}$

> Split the fraction into two powers of x first.

Example 02

Video · E02

Let $f(x) = x^3 + 3x^2 - 9x - 2$.
(a) Find $f'(x)$.
(b) Find the points with a horizontal tangent.
(c) Write the tangent line at $x = 0$.
(d) Find the other x with a parallel tangent.

> Horizontal means slope 0. Parallel means the same slope.

Example 03

Video · E03

Let $f(x) = \begin{cases} ax^2 + 1, & x \le 2 \\ bx - 3, & x > 2 \end{cases}$, where a and b are constants.
Find a and b so that f is differentiable at $x = 2$. Justify your answer.

> Two unknowns need two conditions.

VIDEO COMPLETE Check your example answers at the end of this booklet.

2.5–2.6 / Example answers

The Power Rule and the Sum Rules

E01 (D)

E02 (a) $f'(x) = 3x^2 + 6x - 9$; (b) $(-3, 25)$ and $(1, -7)$; (c) $y = -2 - 9x$; (d) -2

E03 (a) $a = 1$; (b) $b = 4$