2.4 / Practice workbook

Differentiability and Continuity

Practice

Problems 1–9 / Show your reasoning

1 Let $f(x) = |x^2 - 4|$. (a) Find the left-hand limit $\lim_{h\to 0^-} \dfrac{f(2+h)-f(2)}{h}$. (b) Find the right-hand limit (with $h \to 0^+$). (c) Is f differentiable at $x = 2$? (d) Is f continuous at $x = 2$?

2 The function f is differentiable at $x = 3$, with $f(3) = -2$ and $f'(3) = 5$. (a) Find $\lim_{h\to 0} f(3+h)$. (b) Find $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h}$. (c) Why are the two answers different?

3 Let $f(x) = \begin{cases} x^2, & x < 1 \\ 2x, & x \geq 1. \end{cases}$ A student says: "The left piece has slope 2 at $x = 1$ and the right piece has slope 2, so f is differentiable at $x = 1$." (a) Is f continuous at $x = 1$? (b) Find $\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h}$. (c) Is the student right? Explain.

4 If f is continuous at $x = 3$, which of the following must be true?

A. $f'(3)$ exists.

B. $\lim_{h\to 0}[f(3+h) - f(3)] = 0$

C. $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = 0$

D. $\lim_{h\to 0} f(3+h) = 0$

5 Let $f(x) = \begin{cases} x^2 + 4x + 1, & x < 0 \\ 4x + 1, & 0 \leq x < 2 \text{At} \\ 13 - 2x, & x \geq 2. \end{cases}$ which values of x is f not differentiable?

A. none

B. $x = 2$ only

C. $x = 0$ and $x = 2$ only

D. $x = -2$ and $x = 2$ only

6 Which of the following functions is continuous at $x = 1$ but not differentiable at $x = 1$?

A. $f(x) = \dfrac{x^2 + 8x - 9}{x - 1}$

B. $f(x) = (x-1)\,|x-1|$

C. $f(x) = |3x - 3| + x$

D. $f(x) = (x-1)^2 + 5$

2.4 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \sqrt[3]{x-2} + 1$.

(a) Find $\lim_{h \to 0^+} [f(2+h) - f(2)]$. What does this say about the continuity of f at $x = 2$ from the right?

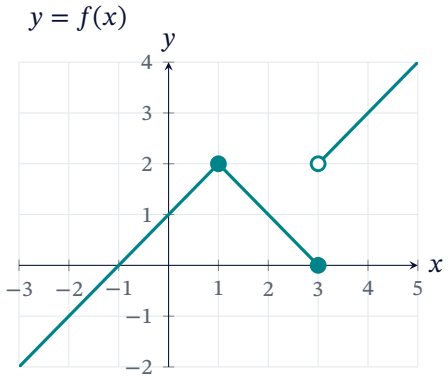
(b) Find $\lim_{h \to 0^+} \dfrac{f(2+h) - f(2)}{h}$.

(c) Is f differentiable at $x = 2$? Describe the graph of f at $x = 2$.

2.4 / AP-style free response

Problem 8 ➤ */ Show the work and the reasons a grader looks for.*

The graph of f is shown.



(a) Find $f'(-1)$.

(b) Is f differentiable at $x = 1$? Give a reason.

(c) Is f differentiable at $x = 3$? Give a reason.

2.4 / AP-style free response

Problem 9 ➤ */ Show the work and the reasons a grader looks for.*

The function f is differentiable for all real numbers. Selected values of f are in the table.

x	0	2	5
$f(x)$	-3	4	1

(a) Explain why f is continuous on $[0, 2]$.

(b) Must there be a value c, $0 < c < 2$, with $f(c) = 0$? Justify your answer.

(c) Must there be a value c, $2 < c < 5$, with $f(c) = 0$? Explain.

2.4 / Answer key

Differentiability and Continuity

1 (a) -4; (b) 4; (c) no: the one-sided limits -4 and 4 are not equal; (d) yes: $f(2+h)-f(2)=|4h+h^2|\to 0$

2 (a) -2: differentiable at 3, so continuous at 3; (b) 5: this limit is the definition of $f'(3)$; (c) (a) is the height $f(3)$ of the graph; (b) is its slope $f'(3)$ there

3 (a) no: $\lim_{x\to 1^-} f(x)=1$ but $f(1)=2$; (b) $\lim_{h\to 0^-}\frac{(1+h)^2-2}{h}=\infty$ (does not exist as a real number); (c) no: f is not continuous at 1, so it is not differentiable there; matching slopes is not enough

4 (B)

5 (B)

6 (C)

7 (a) $\sqrt[3]{h}\to 0$; with the same from the left, f is continuous at $x=2$; (b) $\lim_{h\to 0^+}\frac{\sqrt[3]{h}}{h}=\lim_{h\to 0^+}\frac{1}{\sqrt[3]{h^2}}=\infty$; (c) no: the difference quotient grows without bound, so $f'(2)$ does not exist; the graph has a vertical tangent line at $x=2$

8 (a) 1; (b) no: f is continuous at 1, but the slope from the left is 1 and from the right is -1 (a corner); (c) no: $\lim_{x\to 3^+} f(x)=2$ but $f(3)=0$, so f is not continuous at 3, so not differentiable

9 (a) f is differentiable, so it is continuous; (b) yes: $f(0)=-3<0<4=f(2)$, IVT; (c) not necessarily: 0 is not between $f(2)=4$ and $f(5)=1$