2.4 / Differentiability and Continuity

Watch the lesson ➤

Determining when derivatives do and do not exist

Learning goal

Decide where a function is or is not differentiable, use "differentiable, so continuous" as a reason, and show it with the h form.

Key ideas

01 f is **differentiable** at $x = c$ when $f'(c) = \lim_{h\to 0} \dfrac{f(c+h) - f(c)}{h}$ is a real number: both one-sided limits exist and are equal.

02 f is continuous at $x = c$ when $\lim_{h\to 0}[f(c+h) - f(c)] = 0$ (the **h form** of continuity).

03 If f is differentiable at c, then f is **continuous** at c: $f(c+h) - f(c) = \dfrac{f(c+h) - f(c)}{h} \cdot h \to f'(c) \cdot 0 = 0$.

04 The converse is **false**. f can be continuous at c and still not differentiable: a corner, a cusp, or a vertical tangent line.

05 If f is not continuous at c (a hole, a jump, an asymptote), then f is **not** differentiable at c.

06 At a joint of a piecewise function, find each one-sided limit of the difference quotient with $f(c)$ and its own **piece**.

Remember

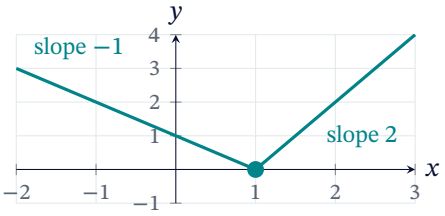
differentiable
$f'(c)$ exists as a real number.

corner
A point where the slopes from the left and from the right are different.

vertical tangent line
The tangent line is vertical: the difference quotient grows without bound, so $f'(c)$ does not exist.

cusp
A sharp point where the two sides become vertical, one going up and one going down.

Continuous, but not differentiable



At $x = 1$ the graph has no break: f is continuous. The slope from the left is -1 and from the right is 2, so the difference quotient has no single limit: $f'(1)$ does not exist.

Class notes

2.4 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Give the reason in words before using continuity: "R is differentiable, so R is continuous." (2025 AB3(b))	Do not write only "R is continuous" when a theorem needs it: that sentence alone did not earn the point.
At a joint, show both one-sided limits of the difference quotient, each with $f(c)$.	Do not argue "continuous, so differentiable". A corner is continuous but not differentiable.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Continuous is not enough

Code D6

✗ $f(x) = |x^2 - 4|$ has no break at $x = 2$, so f is differentiable at $x = 2$.

✓ The one-sided limits of the difference quotient are -4 and 4: a corner. Continuous, but not differentiable.

02 Matching slopes is not enough

Code D8

✗ x^2 and $2x$ both have slope 2 at $x = 1$, so f is differentiable at $x = 1$.

✓ First check continuity: the left side heads to 1 but $f(1) = 2$. Not continuous, so not differentiable.

03 A smooth turning point is differentiable

Code D7

✗ The graph turns at $x = 0$, so $f'(0)$ does not exist.

✓ A smooth top or bottom has a flat tangent line: $f'(0) = 0$. Only corners, cusps, vertical tangents, and breaks fail.

Real AP questions on this topic Free on AP Central. Try them after the practice.

- 2025 AB FRQ 3(b) – "differentiable, so continuous" point mean 0.27
- 2023 AB FRQ 1(b) – continuity from differentiability as a theorem condition

2.4 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

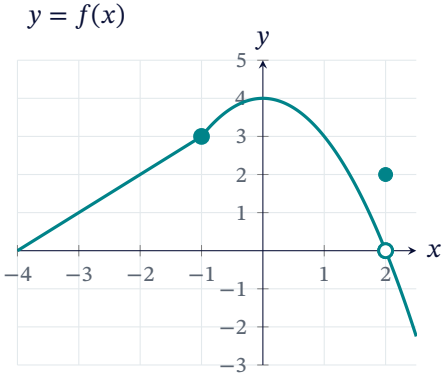
Type	Method	Problems
Where f is not differentiable, read from a graph	Not differentiable at a corner, a cusp, a vertical tangent line, or any break (hole, jump, asymptote). A smooth turning point has slope 0: it is differentiable.	E01 · Practice 8
Differentiable at a joint of a piecewise function, by one-sided difference quotients	(i) Is f continuous at c? If not, stop: not differentiable. (ii) Left and right limits of $\frac{f(c+h)-f(c)}{h}$, each with its own piece and $f(c)$. (iii) Equal real numbers means differentiable.	E02 · Practice 3, 5
Differentiable implies continuous — the reason in words and in the h form	$f(c+h)-f(c)=\frac{f(c+h)-f(c)}{h}\cdot h \to f'(c)\cdot 0=0$, so f is continuous at c. Write: "f is differentiable, so f is continuous."	E03 · Practice 2, 9
Continuous does not mean differentiable	Look for a corner (one-sided slopes differ) or a vertical tangent line (the difference quotient grows without bound).	Practice 1, 4, 6, 7

Example 01

Video · E01

The graph of f is shown for $-4 < x < 2.5$. At which values of x is f not differentiable?

$y = f(x)$



A. $x = -1$ only
B. $x = 2$ only
C. $x = -1$ and $x = 2$ only
D. $x = -1$, $x = 0$, and $x = 2$

Look for a corner, a break, a cusp, or a vertical tangent.

Example 02

Video · E02

Let $f(x) = \begin{cases} 3x - 1, & x \le 2 \\ x^2 - 2x + 5, & x > 2 \end{cases}$

(a) Show that f is continuous at $x = 2$.

(b) Find $\lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h}$ and $\lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h}$.

(c) Is f differentiable at $x = 2$? Give a reason.

> Each side uses its own piece, and both use f(2).

Example 03

Video · E03

g is differentiable for all real numbers. Selected values of g are in the table.

(a) Use $\lim_{h \to 0}[g(3 + h) - g(3)]$ to show g is continuous at $x = 3$.

(b) Must $g(c) = 6$ for some c in $(1, 3)$? Justify.

x	1	3	5
$g(x)$	9	4	8

> Write the difference as the difference quotient times h.

VIDEO COMPLETE Check your example answers at the end of this booklet.

2.4 / Example answers

Differentiability and Continuity

E01 (C)

E02 (a) $f(2) = 5$ and both one-sided limits of f are 5, so $\lim_{h\to 0}[f(2+h) - f(2)] = 0$; (b) left: 3; (c) right: 2; (d) no: the one-sided limits 3 and 2 are not equal (a corner)

E03 (a) $g(3+h) - g(3) = \frac{g(3+h)-g(3)}{h} \cdot h \to g'(3) \cdot 0 = 0$, so g is continuous at $x = 3$; (b) yes: g is differentiable, so g is continuous on $[1, 3]$; $g(3) = 4 < 6 < 9 = g(1)$; by the Intermediate Value Theorem there is at least one such c