2.3 / Practice workbook

Estimating Derivatives from Tables and Graphs

Practice

Problems 1–9 / Show your reasoning

1 Selected values of a differentiable function f are given in the table. (a) Estimate $f'(4)$ using the table values closest to $x = 4$ on either side. (b) Estimate $f'(2)$.

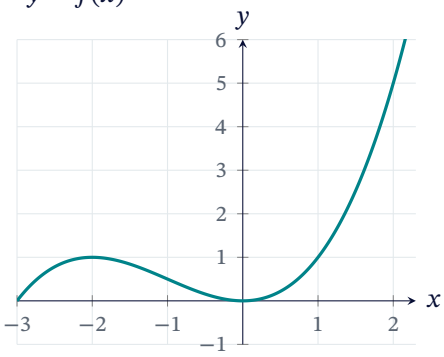
x	1	3	4	6	9
$f(x)$	2	7	10	13	22

2 The temperature of a pie taken out of an oven is $T(t)$ degrees Fahrenheit, where t is measured in minutes. Selected values are given in the table. A student estimates $T'(12)$, in degrees Fahrenheit per minute, this way: $T'(12) \approx \dfrac{262 - 236}{15 - 10} = 5.2$.
(a) What is wrong with the work? (b) Give a correct estimate of $T'(12)$.

t (minutes)	0	5	10	15	20
$T(t)$ (°F)	350	300	262	236	220

3 The graph of a differentiable function f is shown. (a) Use the points on the graph at $x = -2$ and $x = 0$ to estimate $f'(-1)$. (b) Use the points at $x = 0$ and $x = 2$ to estimate $f'(1)$.

$y = f(x)$



4 Selected values of a differentiable function R are in the table. What is the best estimate of $R'(7)$?

t	0	2	5	9	10
$R(t)$	40	46	55	52	49

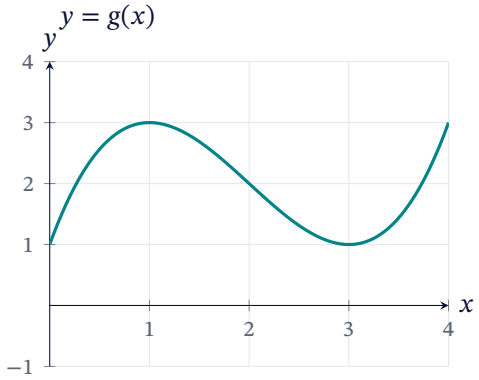
A. $\frac{3}{4}$
B. 53.5
C. $-\frac{3}{4}$
D. $-\frac{6}{5}$

5 A cyclist's velocity is $v(t)$ meters per second at time t seconds. Selected values are in the table. What is the best estimate of $v'(6)$, with units?

t (seconds)	0	4	8	12
$v(t)$ (m/s)	3	7	13	14

A. 1.5 meters per second
B. 1.5 meters per second per second
C. 1.5 meters
D. 10 meters per second

6 The graph of a differentiable function g for $0 \le x \le 4$ is shown. Which of the following are true?
I. $g'(2) < 0$
II. $g'(3) < g'(2)$
III. The average rate of change of g over $[1, 3]$ is -1.



A. I only
B. I and II only
C. I and III only
D. I, II, and III

2.3 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

A museum has $M(t)$ people inside, t hours after it opens (M is differentiable). Selected values are in the table.

t (hours)	0	1	3	4	7	8
$M(t)$ (people)	20	95	160	170	140	90

(a) Estimate $M'(2)$. Show the work and units.

(b) Explain (a) in context.

(c) Estimate $M'(7.5)$. Based on it, is M increasing or decreasing? Why?

2.3 / AP-style free response

Problem 8 ➤ */ Show the work and the reasons a grader looks for.*

Water flows out of a tank at $r(t) = \dfrac{12}{1 + \frac{t}{10}}$ gallons per minute, t in minutes.

t (minutes)	28	30	32
$r(t)$	3.1579	3.0000	2.8571

(a) Estimate $r'(30)$. Show the work and units.

(b) A calculator gives $r'(30) = -0.075$. Explain it in context.

(c) Is r increasing or decreasing at $t = 30$? Why?

2.3 / AP-style free response

Problem 9 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \sqrt{x+5}$ and $g(x) = 0.3x^2$. For $0 \le x \le 3$, the graph of f is above the graph of g. Let $D(x)$ be the vertical distance between the two graphs. A calculator may be used.

(a) Find the rate of change of D with respect to x at $x = 2$. Show the setup and give your answer to three decimal places.

(b) Is the vertical distance increasing or decreasing at $x = 2$? Give a reason.

2.3 / Answer key

Estimating Derivatives from Tables and Graphs

1 (a) 2; (b) $\frac{5}{2}$

2 (a) the top is $T(10)-T(15)$ but the bottom is $15-10$; the orders must match, so the sign is wrong; (b) $\frac{T(15)-T(10)}{15-10} = \frac{236-262}{5} = -5.2$ degrees Fahrenheit per minute

3 (a) $-\frac{1}{2}$; (b) $\frac{5}{2}$

4 (C)

5 (B)

6 (C)

7 (a) $\frac{M(3)-M(1)}{3-1} = \frac{160-95}{2} = 32.5$ people per hour; (b) at $t = 2$ hours after opening, the number of people in the museum is increasing at a rate of about 32.5 people per hour; (c) $\frac{M(8)-M(7)}{8-7} = -50$ people per hour; decreasing, because the estimate is negative

8 (a) $\frac{r(32)-r(28)}{32-28} = \frac{2.8571-3.1579}{4} \approx -0.0752$ gallons per minute per minute; (b) at $t = 30$ minutes, the rate at which water flows out is decreasing at 0.075 gallon per minute per minute; (c) decreasing, because $r'(30) = -0.075 < 0$

9 (a) $D(x) = \sqrt{x+5} - 0.3x^2$, $D'(2) \approx -1.011$; (b) decreasing, because $D'(2) \approx -1.011 < 0$