Lecture Notes · Unit 2 · Lecture 3 AP CED 2.3

Estimating derivatives at a point

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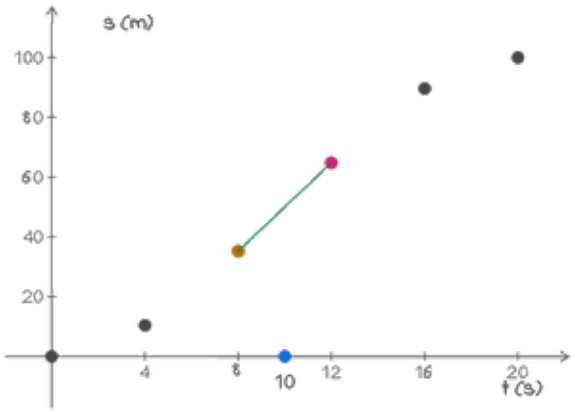
INTRO: NO FORMULA, JUST A TABLE

No formula, just a table

Sam, 100 m in 20 s: a stopwatch table

t (s)	0	4	8	12	16	20
s (m)	0	10.4	35.2	64.8	89.6	100



Between dots → unknown

How fast at t = 10 s?

10 not in the table → can't finish the limit

Closest average: one on each side of 10 → 8, 12

$$s'(10) \approx \frac{s(12)-s(8)}{12-8}$$

$$= \frac{64.8-35.2}{12-8} = \frac{29.6}{4} = 7.4 \text{ m/s}$$

WHY THE CLOSEST PAIR

Why the closest pair?

Far: 0, 20 → $\frac{s(20)-s(0)}{20-0} = \frac{100-0}{20} = 5$

4, 16 → $\frac{s(16)-s(4)}{16-4} = \frac{89.6-10.4}{12} = 6.6$

Close: 8, 12 → 7.4 5 → 6.6 → 7.4

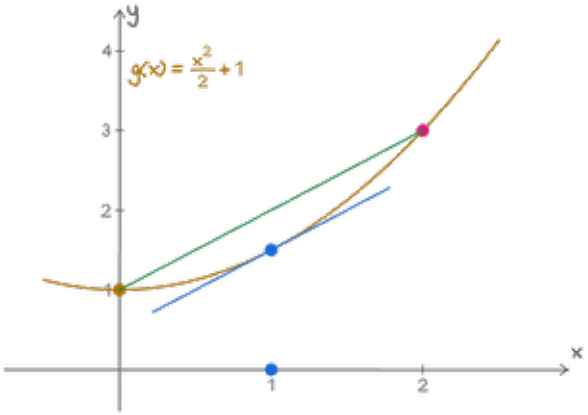
Model: $s(t) = 100\left[3\left(\frac{t}{20}\right)^2 - 2\left(\frac{t}{20}\right)^3\right]$

Calculator, numerical derivative: $s'(10) = 7.500$ m/s

Closer pair → better estimate (usually)

WHY ONE ON EACH SIDE

Why one on each side?



Test: $g(x) = \frac{x^2}{2} + 1$ around $x = 1$

(0, 1) and (2, 3) → one on each side of 1

$$\frac{g(2)-g(0)}{2-0} = \frac{3-1}{2} = 1$$

Tangent at 1 → slope 1 → parallel

Parabola + same distance each side → exact

Sam: 7.4 vs 7.5 → close

Both sides? → one on each side

THE RECEIPT AND THE UNITS

Write the whole answer

✗ $s'(10) \approx 7.4$ → from where?

✓ $s'(10) \approx \frac{s(12)-s(8)}{12-8} = \frac{64.8-35.2}{12-8} = 7.4$

Receipt = difference quotient with the table's numbers

Stop at the rate: 7.4 m/s

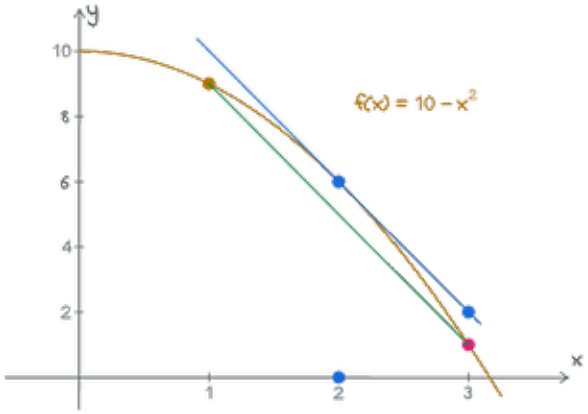
Units: $\frac{\text{units of } s}{\text{units of } t}$

$\frac{64.8\text{ m} - 35.2\text{ m}}{12\text{ s} - 8\text{ s}} = \frac{29.6\text{ m}}{4\text{ s}} = 7.4\text{ m per s}$

v in m/s → v' in m/s per s

ONLY A GRAPH: TWO WAYS

Only a graph



Practice: $f(x) = 10 - x^2$ → estimate $f'(2)$

Curve only → two readable points, one on each side

$f'(2) \approx \frac{f(3) - f(1)}{3 - 1} = \frac{1 - 9}{2} = -4$

Tangent drawn → points on the tangent

$(2,6) \to (3,2)$ $\frac{2 - 6}{3 - 2} = \frac{-4}{1} = -4$

CALCULATOR: THE NUMBER; YOU: THE SENTENCE

A calculator gives a number

Calculator → the number only

Candle: height $H(t)$ cm, t hours; $H'(3) = -1.2$

In four pieces

Time: $t = 3$ h

What: candle height

Up or down: is decreasing

Rate: 1.2 cm per hour

PROBLEM 1 OF 2

Snow on the roof

The snow depth is $D(t)$ centimeters at time t hours. Selected values are in the table.

(a) Estimate $D'(4)$. Show the work, with units.

(b) Estimate $D'(10.5)$.

(c) Explain the meaning of (b).

t	0	2	5	6	9	12
$D(t)$	8	11	17	18	21	20

Solution (a) The two closest times

(a)

Plan: the two closest times, one on each side of 4

4 between 2 and 5 (not the middle)

$D'(4) \approx$

$\frac{D(5) - D(2)}{5 - 2}$

$= \frac{17 - 11}{5 - 2} = \frac{6}{3}$

$\frac{6\text{ cm}}{3\text{ hours}} = 2\text{ cm per hour}$

(b) Your turn: $D'(10.5)$

Solution (b) and (c): the rate and its meaning

t	0	2	5	6	9	12
$D(t)$	8	11	17	18	21	20

(b)

10.5 between 9 and 12

$D'(10.5) \approx$

$\frac{D(12) - D(9)}{12 - 9}$

$= \frac{20-21}{12-9} = \frac{-1}{3}$

$\frac{-1 \text{ cm}}{3 \text{ hours}} = -\frac{1}{3}$ cm per hour

(c)

Negative → depth going down

At t = 10.5 hours, the snow depth

is decreasing at about $\frac{1}{3}$ cm per hour.

Solution (c) A common mistake: two minuses

t	0	2	5	6	9	12
D(t)	8	11	17	18	21	20

(c)

✗ Decreasing at a rate of $-\frac{1}{3}$

✓ Decreasing at about $\frac{1}{3}$ cm per hour

Say the minus once

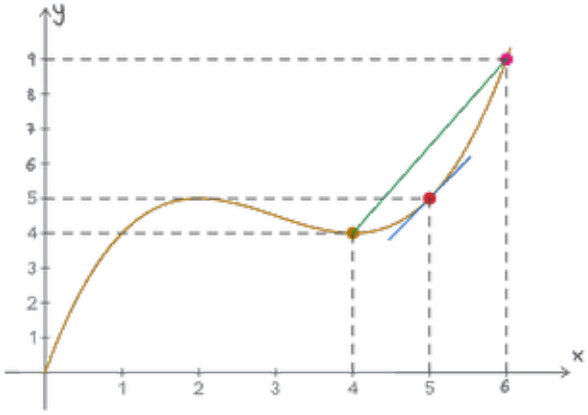
PROBLEM 2 OF 2

Graph only

The graph of a differentiable function f is shown.

Which is closest to $f'(5)$?

(A) $\frac{5}{2}$ (B) 5 (C) $-\frac{5}{2}$ (D) 1



Solution Two points on the curve, one on each side

Plan: grid points on the curve, one on each side of 5

$f'(5) \approx$

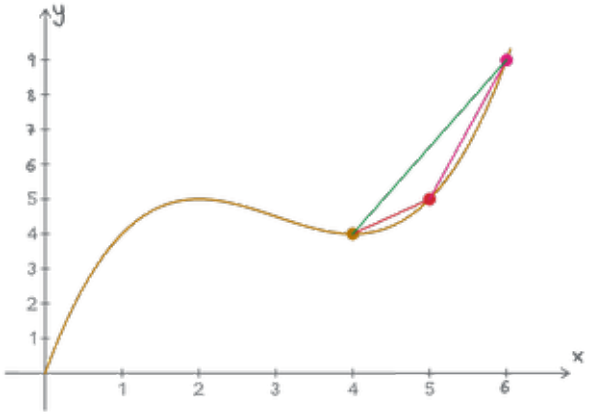
$\frac{f(6)-f(4)}{6-4} = \frac{9-4}{6-4} = \frac{5}{2}$

So (A) $\frac{5}{2}$

(B) = f(5), the height, not the slope

(C) $-\frac{5}{2}$, but rising at 5 → slope > 0

Solution (D): one side only



(D) $1 = \frac{f(5) - f(4)}{5 - 4}$

Right side: $\frac{f(6) - f(5)}{6 - 5} = \frac{9 - 5}{1} = 4$

Curve bends: left secant 1, right 4 → each side $\frac{5}{2}$

Both sides in the data? → one on each side

B, C, D out → (A)

WRAP-UP

No formula: what we do

About 7.4 m/s, with ≈

Structure first, then numbers: the receipt

Units of s over units of t

Meaning: when, what, up or down, how fast

one on each side, when both sides are there

Workbook → video

Next → 2.4 → where f′ does not exist

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.