2.2 / The Derivative Function and the Tangent Line

Watch the lesson ➤

Defining the derivative of a function and using derivative notation

Learning goal
Find the derivative as a function, write it in derivative notation, and use $f'(a)$ as the slope of the tangent line.

Key ideas

01 With x in place of a, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ is a new **function**. Its value at $x = a$ is the number $f'(a)$.

02 For $y = f(x)$, the derivative is also written $\dfrac{dy}{dx}$ or y'. The number $f'(a)$ is $\left.\dfrac{dy}{dx}\right|_{x=a}$, read "$dy/dx$ **at** $x = a$."

03 $f'(a)$ is the **slope** of the line tangent to the graph of f at $x = a$.

04 The tangent line goes through the **point** $(a, f(a))$: $y = f(a) + f'(a)(x - a)$.

05 A derivative can be shown in a graph (a slope), in numbers, in a formula, or in **words** (a rate with units).

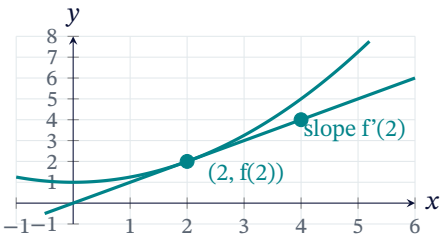
Remember

derivative function
The function f' whose value at each x is the slope of the tangent line there.

tangent line
The line through $(a, f(a))$ with slope $f'(a)$.

$\dfrac{dy}{dx}$
Another way to write the derivative of $y = f(x)$. Read it "d y d x."

The tangent line uses the point and the slope



At $x = 2$ the point is $(2, f(2)) = (2, 2)$ and the slope is $f'(2) = 1$, so the tangent line is $y = 2 + 1(x - 2)$. Over a run of 2 the line rises 2.

Class notes

2.2 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the derivative at a point as a number at that point: $f'(2) = 9$.	Do not write $f'(x) = 9$ when you mean the number $f'(2) = 9$.
Write the tangent line from the point and the slope. You do not need to simplify: $y = 2 + 9(x - 2)$.	Do not use the expression $f'(x)$ as the slope of a tangent line. Use the number $f'(a)$.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 A tangent line needs the point

Code D5

✗ $f(-1) = 4$ and $f'(-1) = -3$, so the tangent line is $y = -3(x + 1)$.

✓ Use the point $(-1, 4)$ too: $y = 4 - 3(x + 1)$.

02 The height is not the slope

Code D1

✗ The tangent line at $x = 1$ goes through $(1, 3)$, so $f'(1) = 3$.

✓ 3 is $f(1)$, the height. $f'(1)$ is the slope of the line: rise over run.

Real AP questions on this topic Free on AP Central. Try them after the practice.

Tangent lines appear inside larger released FRQs, such as 2024 AB5(a) and 2023 AB3(b), usually with the slope given by a derivative the student already has. No released FRQ from 2023–2026 asks for a derivative from its definition.

2.2 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
$f'(x)$ as a function from the definition	Use x in place of a: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. Then $f'(a)$ is that function at a.	E02 · Practice 1
Derivative notation, and facts in words written as symbols	"The rate of change of y with respect to x at $x = a$" is $f'(a) = \left.\frac{dy}{dx}\right\|_{x=a}$.	Practice 2
The tangent line at a point	Point $(a, f(a))$, slope $f'(a)$: $y = f(a) + f'(a)(x - a)$.	E01, E03 · Practice 4, 6, 7
$f'(a)$ read from a graph	$f'(a)$ is the slope of the tangent line: rise over run between two grid points on that line.	Practice 3
What must be true when the derivative limit is known	The two forms are the same number $f'(a)$, and it is the slope of the tangent line. It says nothing about the value $f(a)$.	Practice 5

Example 01

Video · E01

Let $f(x) = x^3 - 3x$.
(a) Use the definition of the derivative to find $f'(2)$.
(b) Write an equation of the line tangent to the graph of f at $x = 2$.

> A line needs a point and a slope.

Example 02

Video · E02

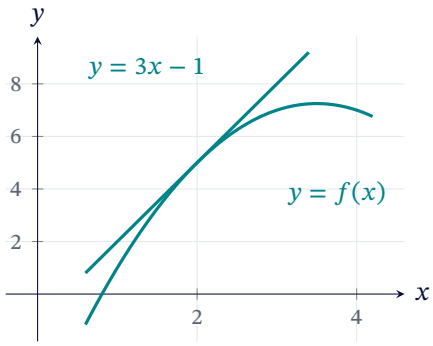
Let $f(x) = 2x^2 - x + 3$. By the definition, $f'(x) = 4x - 1$.
(a) Find the point where the tangent is parallel to $y = 7x$.
(b) Write an equation of that tangent line.

> Parallel lines have the same slope.

Example 03

Video · E03

The line $y = 3x - 1$ is tangent to the graph of f at $x = 2$.
(a) Find $f(2)$.
(b) Find $f'(2)$.
(c) Find $\lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$.



> Plan: at $x = 2 \rightarrow$ same height, same slope

VIDEO COMPLETE Check your example answers at the end of this booklet.

2.2 / Example answers

The Derivative Function and the Tangent Line

E01 (a) 9; (b) $y = 2 + 9(x - 2)$

E02 (a) $(2, 9)$; (b) $y = 9 + 7(x - 2)$

E03 (a) 5; (b) 3; (c) 3