2.1 / Practice workbook

The Rate of Change at an Instant

Practice

Problems 1–7 / Show your reasoning

1 Let $f(x) = \dfrac{x}{x+2}$. Use $\lim_{x\to 2} \dfrac{f(x) - f(2)}{x - 2}$ to find $f'(2)$.

2 Let $f(x) = 5x - x^2$. A student finds $f'(2)$ this way: $\lim_{h\to 0} \dfrac{f(2) - f(2+h)}{h} = \lim_{h\to 0} \dfrac{-h + h^2}{h} = -1$.
(a) What is wrong with the setup? (b) Find the correct value of $f'(2)$.

3 The cost of making x chairs is $C(x) = 300 + 12x - 0.02x^2$ dollars, $0 \le x \le 250$. (a) Find the average rate of change of C over $100 \le x \le 200$. (b) Use a limit to find the instantaneous rate of change of C at $x = 100$. Include units in both answers.

4 The limit $\lim_{h\to 0} \dfrac{(2+h)^3 - 8}{h}$ is $f'(a)$ for which function f and number a?

A. $f(x) = x^3,\ a = 8$
B. $f(x) = x^3,\ a = 2$
C. $f(x) = (2+x)^3,\ a = 2$
D. $f(x) = x^3 - 8,\ a = 0$

5 The amount of water in a tank is $W(t)$ liters, where t is the time in hours. Which of the following best explains the statement $W'(5) = -3$?

A. During the first 5 hours, the amount of water decreased by 3 liters.

B. At $t = 5$ hours, the amount of water is decreasing at a rate of 3 liters per hour per hour.

C. At $t = 5$ hours, the amount of water is decreasing at a rate of 3 liters per hour.

D. From $t = 4$ to $t = 5$ hours, the amount of water decreased by exactly 3 liters.

2.1 / AP-style free response

Problem 6 ➤ / *Show the work and the reasons a grader looks for.*

The radius of a circular oil spill is $r(t) = 2\sqrt{t}$ centimeters, where t is the time in seconds, $t > 0$.

(a) Find the average rate of change of r over the interval $1 \le t \le 4$. Indicate units of measure.

(b) Write a limit expression for $r'(4)$, and evaluate it.

(c) Your answers to (a) and (b) are different. Explain why, using the words average and instant.

2.1 / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

Selected values of a function f are given in the table.

x	0	1	2	4	6
$f(x)$	3	5	4	10	7

(a) Find the average rate of change of f over the interval $1 \le x \le 4$.

(b) Let $k(x) = f(2x)$. Find the average rate of change of k over the interval $1 \le x \le 2$.

(c) Let $g(x) = x^2 - 6x + 2$. Find the value of $b > 1$ for which the average rate of change of g over the interval $1 \le x \le b$ is 0.

2.1 / Answer key

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1 $\frac{1}{8}$

2 (a) the top is $f(2) - f(2+h)$; it must be $f(2+h) - f(2)$, the same order as the change $h = (2+h) - 2$ on the bottom, so the sign flips; (b) 1

3 (a) 6 dollars per chair; (b) 8 dollars per chair

4 (B)

5 (C)

6 (a) $\frac{2}{3}$ centimeter per second; (b) $\lim_{h \to 0} \frac{2\sqrt{4+h}-4}{h} = \frac{1}{2}$; (c) (a) is an average over $1 \le t \le 4$ (two times); (b) is the rate at the instant $t = 4$ (one time, a limit)

7 (a) $\frac{f(4) - f(1)}{4 - 1} = \frac{10 - 5}{3} = \frac{5}{3}$; (b) $\frac{k(2) - k(1)}{2 - 1} = \frac{f(4) - f(2)}{1} = 10 - 4 = 6$; (c) $\frac{g(b) - g(1)}{b - 1} = \frac{b^2 - 6b + 5}{b - 1} = b - 5 = 0$, so $b = 5$