2.1 / The Rate of Change at an Instant

Watch the lesson ➤

Defining average and instantaneous rates of change at a point

Learning goal

Find the rate of change at an instant as the limit of average rates of change, name it $f'(a)$, and explain it with units.

Key ideas

01 The **difference quotient** $\dfrac{f(a+h)-f(a)}{h}$, $h \neq 0$, is the average rate of change of f from a to $a+h$. So is $\dfrac{f(x)-f(a)}{x-a}$, $x \neq a$.

02 The **derivative** of f at $x=a$ is $f'(a)=\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$, if the limit exists. It is the instantaneous rate of change at $x=a$.

03 The same number is $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$. Both are **forms** of the definition of $f'(a)$.

04 **Square minus triangle**: in $\dfrac{f(\square)-f(\triangle)}{\text{bottom}}$ the bottom must be $\square-\triangle$ worked out. If the top goes to $f(a)-f(a)$, the limit is $f'(a)$.

05 The **units** of $f'(a)$ are the units of f per unit of x.

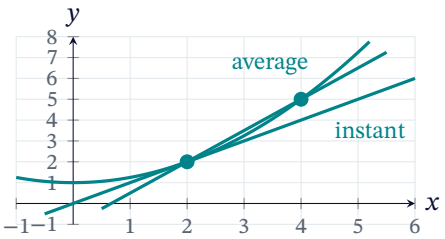
Remember

derivative
The instantaneous rate of change $f'(a)$: the limit of average rates of change as the interval shrinks to a.

difference quotient
A change in f divided by the change in x: an average rate of change.

rate at an instant
The instantaneous rate of change: the rate at one value of x, not over an interval.

Shrink the interval, and the average becomes the rate at an instant



The line through $(2,2)$ and $(4,5)$ has slope $\frac{3}{2}$, the average rate of change on $[2,4]$. As the second point slides to $x=2$, the slope approaches $f'(2)$, the rate of change at the instant $x=2$.

Class notes

2.1 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Give a rate with units of f per unit of x: "$V'(1) = 4$ liters per minute."	Do not turn a rate into a value of the function. When the question asks for $f'(a)$, stop at the rate.
Keep lim on every line until h cancels and you substitute: $\lim_{h\to 0} \frac{4h-h^2}{h} = \lim_{h\to 0}(4-h) = 4$.	Do not give an average over an interval when the question asks for the rate at an instant.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Same order on top and bottom

Code D2

✗ $\frac{f(2)-f(2+h)}{h}$: the top is earlier minus later, the bottom is later minus earlier. The sign flips.

✓ $\frac{f(2+h)-f(2)}{h}$: later minus earlier on both.

02 A rate needs its units

Code D3

✗ $V'(1) = 4$ liters.

✓ $V'(1) = 4$ liters per minute: units of V per unit of t.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 asks for a derivative from its definition. The 2012 Course Description AB multiple-choice question 1 asks which derivative a limit represents (the type of E02 and the first AP multiple-choice practice problem). A rate with units is asked every year in the first part of a table FRQ, such as 2025 AB3(a).

2.1 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Average rate of change on an interval and the rate at an instant, with units	Average: the difference quotient on the interval, a number in units of f per unit of x. Instant: the limit of that quotient as $h \to 0$.	E01 · Practice 3, 7
Which limit is $f'(a)$, and read f and a from a limit	Match the shape $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$: the same a in both places, the same order on top and bottom, h alone on the bottom.	E02 · Practice 4
$f'(a)$ from the definition	Write the limit, simplify the top until h (or $x-a$) cancels, then substitute.	Practice 1, 2
The meaning of $f'(a)$ in context, with units	At [the time], [the quantity] is increasing (decreasing) at [the size] [units of f] per [unit of x].	Practice 5, 6

Example 01

Video · E01

The volume of water in a tank is $V(t) = 40+6t-t^2$ liters, t in minutes, $0 \le t \le 6$.
(a) Find the average rate of change of V over $1 \le t \le 3$, with units.
(b) Write a limit for the instantaneous rate of change at $t = 1$. Evaluate it, with units.

Average uses two times. An instant uses a limit.

Example 02

Video · E02

Let $f(x) = 3x^2 + 2x$. Which is the instantaneous rate of change of f at $x = -1$?

A. $\lim_{h\to 0} \dfrac{f(-1+h) - f(-1)}{-1+h}$

B. $\lim_{h\to 0} \dfrac{f(-1+h) - f(-1)}{h}$

C. $\lim_{h\to 0} \dfrac{f(h) - f(-1)}{h}$

D. $\lim_{x\to -1} \dfrac{f(x) - f(-1)}{-1-x}$

> Check the bottom, the order, and where each f is taken.

VIDEO COMPLETE Check your example answers at the end of this booklet.

2.1 / Example answers

The Rate of Change at an Instant

E01 (a) 2 liters per minute; (b) $\lim_{h \to 0} \frac{V(1+h)-V(1)}{h} = 4$ liters per minute

E02 (B)