1.16 / Practice workbook

Working with the Intermediate Value Theorem

Practice

Problems 1–7 / Show your reasoning

1 The function f is continuous on $[0, 5]$. Selected values of f are given in the table. What is the least number of solutions of $f(x) = 1$ on $[0, 5]$?

x	0	1	2	3	4	5
$f(x)$	4	0	1	3	-2	2

A. 2
B. 3
C. 4
D. 5

1.16 / AP-style free response

Problem 2 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} k - x, & x < 2 \\ x^2 - k, & x \geq 2 \end{cases}$, where k is a constant.

(a) Find k so that f is continuous at $x = 2$.

(b) With this k, find $f(0)$ and $f(3)$.

(c) With this k, explain why $f(c) = 5$ for at least one c in $(0, 3)$.

3 For which of the following functions f does the Intermediate Value Theorem guarantee that the equation $f(x) = 0$ has at least one solution in the interval $(0, 1)$?

A. $f(x) = \dfrac{1}{2x - 1}$

B. $f(x) = (2x - 1)^2$

C. $f(x) = e^x - 2$

D. $f(x) = x^2 - 2x$

1.16 / AP-style free response

Problem 4 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = 2\sin x$ and $g(x) = x - 1$. Let $h(x) = f(x) - g(x)$.

(a) Explain why h is continuous on $[0, \pi]$.

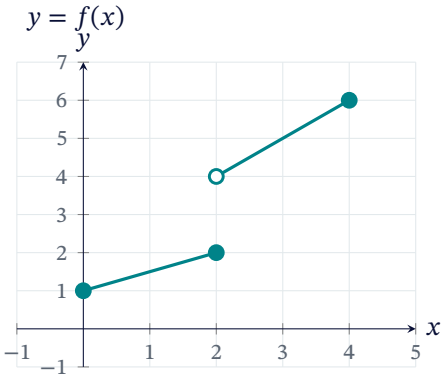
(b) Find $h(0)$ and $h(\pi)$.

(c) Explain why there must be a value c, with $0 < c < \pi$, such that $f(c) = g(c)$.

1.16 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

The graph of f on $[0, 4]$ is shown.



(a) Does the Intermediate Value Theorem guarantee a c in $(0, 4)$ with $f(c) = 3$? Explain.

(b) Is there a c in $(0, 4)$ with $f(c) = 3$? Explain.

(c) Is there a c in $(0, 4)$ with $f(c) = 5$? Explain.

6 The amount of fuel in a tank, $F(t)$ liters, is modeled by a continuous function of time t, in hours. Selected values of $F(t)$ are given in the table. Which of the following must be true?

t (hours)	0	2	4	6	8
$F(t)$ (liters)	40	28	33	21	25

A. $F(t) = 30$ at least once in $(6, 8)$
B. $F(t) = 30$ exactly once in $(0, 4)$
C. $F(t) = 24$ at least once in $(4, 8)$
D. $F(t) = 45$ at least once in $(0, 8)$

1.16 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{x^2 + 4}{x + 1}$.

(a) Explain why f is continuous on $[0, 3]$.

(b) Find $f(0)$, $f(1)$, and $f(3)$.

(c) Explain why the equation $f(x) = 3$ has at least two solutions in $(0, 3)$.

1.16 / Answer key

Working with the Intermediate Value Theorem

1 (C)

2 (a) 3; (b) $f(0) = 3$ and $f(3) = 6$; (c) f is continuous on $[0, 3]$ (each piece is a polynomial, and at $x = 2$ both sides and $f(2)$ equal 1); $f(0) = 3 < 5 < 6 = f(3)$; so $f(c) = 5$ for at least one c in $(0, 3)$

3 (C)

4 (a) f (sine) and g (a polynomial) are continuous for all x, so their difference h is continuous on $[0, \pi]$; (b) $h(0) = 1$ and $h(\pi) = 1 - \pi$; (c) h is continuous on $[0, \pi]$ and $h(\pi) = 1 - \pi < 0 < 1 = h(0)$, so $h(c) = 0$, that is $f(c) = g(c)$, for at least one c in $(0, \pi)$

5 (a) no: f has a jump at $x = 2$, so f is not continuous on $[0, 4]$; the IVT gives no conclusion, even though $f(0) = 1 < 3 < 6 = f(4)$; (b) no: on $[0, 2]$ the heights go from 1 to 2, and on $(2, 4]$ from just above 4 to 6; the height 3 is skipped; (c) yes: $f(3) = 3 + 2 = 5$, so $c = 3$ (the IVT gave no conclusion, but f still takes the value 5)

6 (C)

7 (a) f is a rational function, and its denominator $x + 1$ is 0 only at $x = -1$, which is not in $[0, 3]$. So f is continuous on $[0, 3]$; (b) $f(0) = 4$, $f(1) = \frac{5}{2}$, and $f(3) = \frac{13}{4}$; (c) f is continuous on $[0, 1]$ and on $[1, 3]$ by (a). $f(1) = \frac{5}{2} < 3 < 4 = f(0)$, so $f(c) = 3$ for at least one c in $(0, 1)$. $f(1) = \frac{5}{2} < 3 < \frac{13}{4} = f(3)$, so $f(c) = 3$ for at least one c in $(1, 3)$. The intervals do not overlap: at least two solutions