1.16 / Working with the Intermediate Value Theorem

Watch the lesson ➤

Working with the Intermediate Value Theorem (IVT)

Learning goal
Use the Intermediate Value Theorem correctly — give the reason for continuity on a closed interval, show the value is between f(a) and f(b), and conclude "at least one c" — and say when the theorem gives no conclusion.

Key ideas

01 Does $x^3 + x - 1 = 0$ have a solution? $f(x) = x^3 + x - 1$: $f(0) = -1$, $f(1) = 1$. The graph has no break and goes from below to above, so it must cross $y = 0$: $f(x) = 0$ has a root c in $(0, 1)$, at least one. In general: f continuous on $[a, b]$ and k between $f(a)$ and $f(b)$, then $f(x) = k$ has a root c in (a, b). The Intermediate Value Theorem, the **IVT**.

02 Why continuous? With the same ends and a jump at $x = 3$, $1 < 4 < 7$, yet f never equals 4: not continuous, no promise. Another break can still give $f(c) = 4$: no promise is not "no c". A break, or k not between: no promise.

03 Three sentences: a theorem, so check its condition first. $f(x) = x^3 - 2x^2 + 3$ on $[1, 3]$: a polynomial, so continuous on $[1, 3]$ (or "f is continuous" is given). Then the numbers: $f(1) = 2$, $f(3) = 12$, $f(1) = 2 < 7 < 12 = f(3)$. So by the IVT, $f(c) = 7$ for some c in $(1, 3)$; in words, $f(x) = 7$ has a root in $(1, 3)$. **Three** parts: reason, numbers, conclusion.

04 Make it $h(x) = 0$, find a root c: $f(x) = k \iff f(x) - k = 0$. Does $\cos x = x$ have a root in $(0, \frac{\pi}{2})$? $\cos x = x \iff \cos x - x = 0$. Let $h(x) = \cos x - x$ and find a root c of $h(x) = 0$. $\cos x$ and x are continuous, so h is continuous on $[0, \frac{\pi}{2}]$. $h(0) = 1 > 0$ and $h(\frac{\pi}{2}) = -\frac{\pi}{2} < 0$, so by the IVT $h(x) = 0$ has a root c in $(0, \frac{\pi}{2})$; at that root, $\cos c = c$.

05 In general, on the picture of $y = \cos x$ and $y = x$: $f(x) = g(x)$ where the graphs meet. Move $g(x)$: $f(x) - g(x) = 0$. Call it $h(x) = f(x) - g(x)$: the meeting point c is a root of $h(x) = 0$. At the left end f is above, so $h > 0$; at the right end f is below, so $h < 0$. h is continuous and its sign changes: the IVT with $k = 0$ gives a **root** c.

06 What the IVT does not say: f continuous on $[0, 3]$ with $f(0) = 2$, $f(1) = -1$, $f(2) = 1$, $f(3) = -2$ crosses between each sign change: at **least** 3 roots, maybe more. Not where, not how many: "at least one". A break means no promise, not "no solution".

Remember

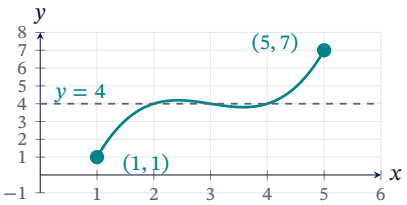
Intermediate Value Theorem (IVT)
A continuous function on $[a, b]$ takes every value between $f(a)$ and $f(b)$ at least once.

closed interval
$[a, b]$: all x from a to b, including both ends.

between
k is between $f(a)$ and $f(b)$ when one value is less than k and the other is greater.

at least one
One or more. The IVT never says "exactly one".

Continuous on $[1, 5]$, and 4 is between $f(1)$ and $f(5)$



f is continuous on $[1, 5]$ and $f(1) = 1 < 4 < 7 = f(5)$, so $f(c) = 4$ for at least one c in $(1, 5)$. Here there are three: $c = 2$, 3, and 4.

Class notes

1.16 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write all three parts: "f is a polynomial, so f is continuous on $[1, 3]$. $f(1) = 2 < 10 < 12 = f(3)$. So there is at least one c in $(1, 3)$ with $f(c) = 10$."	Do not write "f is continuous" without a reason, and do not answer only "yes, IVT."
You do not have to name the theorem. If you name it, name the right one: "by the Intermediate Value Theorem."	Do not list $f(a)$ and $f(b)$ without saying that the value is between them.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Give the reason for continuity first

Code L14

✗ $f(x) = x^3 - 2x$: "$f(1) = -1 < 0 < 4 = f(2)$, so by the IVT, $f(c) = 0$ for some c in $(1, 2)$."

✓ "f is a polynomial, so f is continuous on $[1, 2]$" comes first. Then the values with 0 between them, then "at least one c".

02 Check for a break in the interval

Code L14

✗ $f(x) = (x + 1)/(x - 2)$: "$f(0) = -1/2 < 0 < 4 = f(3)$, so f has a zero in $(0, 3)$."

✓ $f(2)$ is undefined, so f is not continuous on $[0, 3]$: the IVT gives no conclusion. (Here f has no zero in $(0, 3)$; its only zero is $x = -1$.)

03 At least one, not exactly one

Code L14

✗ "$f(0) = 3$ and $f(2) = -1$, and f is continuous, so f has exactly one zero in $(0, 2)$."

✓ The IVT gives at least one zero. f may cross 0 more than once between $x = 0$ and $x = 2$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

- 2025 AB FRQ 3(b) – a table in context: must the function take a given value? AB point means 0.27 (continuity) and 0.28 (conclusion), the lowest of the question
- 2026 AB FRQ 1(d) – the difference of two functions: must it equal 0? "Yes, IVT" alone does not earn the point

1.16 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Show that $f(c) = k$ for some c in (a, b)	Give the reason f is continuous on $[a, b]$: its family, or a fact the problem gives. Compute $f(a)$ and $f(b)$ and show k between them with numbers. Conclude: there is at least one c in (a, b) with $f(c) = k$.	Practice 7
A table in context: must the function take a value?	Continuity is given. Find two times in the interval whose values are on opposite sides of k (not only the endpoints). Write the inequality with the table values and answer with the times and units. If no such pair exists, the IVT gives no conclusion.	E01 · Practice 6
Locate a solution: on which interval must $f(x) = k$ have a solution?	Check that f is continuous on each interval. Compute f at both ends; if k (often 0) is between the two values, $f(c) = k$ for at least one c in that interval.	
Does the IVT apply? Check for a break in $[a, b]$	Check continuity on the whole closed interval first. A hole, a jump, or a vertical asymptote in $[a, b]$: the IVT gives no conclusion, and there may or may not be such a c. With no break, check that k is between $f(a)$ and $f(b)$.	E03 · Practice 3
The value is not between $f(a)$ and $f(b)$	If k is not between $f(a)$ and $f(b)$, the IVT gives no conclusion. It does not say that no c exists; other values of f may still show one.	Practice 5
The least number of solutions	f must be continuous on the whole interval. Mark each pair of neighboring x-values where $f(x) - k$ changes sign strictly: one value below 0, the other above. Each pair gives at least one solution strictly between them. A table value with $f(x) = k$ is a solution too: count it once. The total is the least number, not the exact number.	Practice 1
Two functions: show that $f(c) = g(c)$	Let $h(x) = f(x) - g(x)$. First say why h is continuous: f and g are continuous on $[a, b]$, so their difference h is too. Show $h(a) < 0 < h(b)$ (or the reverse). By the IVT, $h(c) = 0$, so $f(c) = g(c)$, for at least one c in (a, b).	E02 · Practice 4
Choose a constant so that the IVT can be used	Make f continuous at the boundary first: left piece at c = right piece at $c = f(c)$. If no constant works, f is not continuous and the IVT gives no conclusion. Then check that k is between $f(a)$ and $f(b)$.	Practice 2

Example 01

Video · E01

Water depth $D(t)$, in centimeters, is modeled by a continuous function; t is in hours. Must there be a time t, with $0 < t < 8$, at which $D(t) = 45$? Justify.

t (hours)	0	2	3	5	8
$D(t)$ (centimeters)	60	41	30	52	70

> Look at every pair of times, not only $t = 0$ and $t = 8$.

Example 02

Video · E02

f is continuous on $[0, 5]$. Selected values of f are in the table. Must there be a c, with $0 < c < 5$, such that $f(c) = c$? Justify.

x	0	1	3	5
$f(x)$	2	3	1	4

> Move c to one side. Look for a zero of $f(x) - x$.

Example 03

Video · E03

Let $f(x) = \dfrac{x+2}{x-2}$, so $f(1) = -3$ and $f(3) = 5$. Which is true?

A. The IVT guarantees a solution of $f(x) = 0$ in $(1, 3)$.
B. $f(c) = 0$ for some c in $(1, 3)$.
C. The IVT cannot be applied to f on $[1, 3]$.
D. $f(c) = 1$ for some c in $(1, 3)$.

> Check continuity on the whole interval before you look at the values.

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.16 / Example answers

Working with the Intermediate Value Theorem

E01 yes: D is continuous on $[0, 2]$ (given); $D(2) = 41 < 45 < 60 = D(0)$; so $D(t) = 45$ centimeters for at least one t in $(0, 2)$

E02 yes: let $g(x) = f(x) - x$; g is continuous on $[1, 3]$ (f is continuous and x is a polynomial); $g(3) = 1 - 3 = -2 < 0 < 2 = 3 - 1 = g(1)$; so $g(c) = 0$, that is $f(c) = c$, for at least one c in $(1, 3)$

E03 (C)