1.15 / Practice workbook

Limits at Infinity and Horizontal Asymptotes

Practice

Problems 1–7 / Show your reasoning

1

Let $f(x) = \dfrac{ax^2 + 4}{(1 - 2x)(x + 3)}$, where a is a constant. The line $y = 4$ is a horizontal asymptote of the graph of f. What is the value of a?

A. -8

B. -2

C. 4

D. 8

1.15 / AP-style free response

Problem 2 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{\sqrt{16x^2 + 1} - x}{x + 2}$.

(a) Find $\lim_{x \to \infty} f(x)$.

(b) Find $\lim_{x \to -\infty} f(x)$.

(c) Write an equation for each horizontal asymptote of the graph of f.

3

What is $\lim_{x\to\infty} \dfrac{x^5 + 6e^x}{2e^x + \ln x}$?

A. 0
B. $\dfrac{1}{3}$
C. 3
D. ∞

1.15 / AP-style free response

Problem 4 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{3x^2 + 6x}{x^2 + 2}$.

(a) Find $\lim_{x \to \infty} f(x)$.

(b) Find $\lim_{x \to -\infty} f(x)$.

(c) Find the value of x at which the graph of f crosses its horizontal asymptote.

1.15 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

A lake is stocked with fish. Two models for the number of fish t years later are $A(t) = \dfrac{900t + 300}{t + 3}$ and $B(t) = 1000 - 700e^{-\frac{t}{4}}$.

(a) Find $\lim_{t \to \infty} A(t)$.

(b) Find $\lim_{t \to \infty} B(t)$.

(c) In the long run, which model predicts more fish? Interpret its limit in context.

6 Which of the following functions has the line $y = -1$ as a horizontal asymptote?

A. $f(x) = \dfrac{3}{x+1}$

B. $g(x) = \dfrac{x^2}{1-x}$

C. $h(x) = \dfrac{1-x}{2-x}$

D. $p(x) = \dfrac{x}{\sqrt{x^2+9}}$

7 What is $\lim_{x \to -\infty} \dfrac{3x^3 - x}{2 - 6x^2}$?

A. $-\infty$

B. $-\dfrac{1}{2}$

C. 0

D. ∞

1.15 / Answer key

Limits at Infinity and Horizontal Asymptotes

1 (A)

2 (a) 3; (b) -5; (c) $y = 3$ and $y = -5$

3 (C)

4 (a) 3; (b) 3; (c) 1

5 (a) 900; (b) 1000; (c) Model B: in the long run, the number of fish approaches 1000

6 (D)

7 (D)