1.14 / Practice workbook

Infinite Limits and Vertical Asymptotes

Practice

Problems 1–5 / Show your reasoning

1 The graph of which of the following functions has a vertical asymptote at $x = 3$?

A. $f(x) = \dfrac{x^2 - x - 6}{x - 3}$

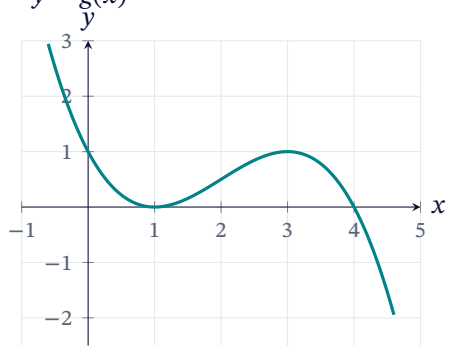
B. $f(x) = \dfrac{x - 3}{x^2 - 4x + 3}$

C. $f(x) = \dfrac{x - 3}{|x - 3|}$

D. $f(x) = \tan\left(\dfrac{\pi x}{6}\right)$

2 The graph of the continuous function g is shown. Let $h(x) = \dfrac{3}{g(x)}$. What are $\lim_{x \to 1} h(x)$ and $\lim_{x \to 4} h(x)$, in that order?

$y = g(x)$



A. ∞ and does not exist

B. ∞ and ∞

C. does not exist and does not exist

D. $-\infty$ and does not exist

1.14 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \ln(9 - x^2)$.

(a) Find the domain of f in interval notation.

(b) Find $\lim_{x \to 3^-} f(x)$.

(c) Find all vertical asymptotes of the graph of f and justify them with limits.

1.14 / AP-style free response

Problem 4 ➤ / *Show the work and the reasons a grader looks for.*

The cost, in thousands of dollars, of removing x percent of a pollutant from a lake is modeled by $C(x) = \dfrac{30x}{100 - x}$ for $0 \le x < 100$.

(a) Find $C(90)$. Include units.

(b) Find $\lim_{x \to 100^-} C(x)$.

(c) Interpret your answer to (b) in context.

1.14 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \dfrac{x + k}{(x - 2)^2}$, where k is a constant.

(a) If $k = 1$, find $\lim_{x \to 2} f(x)$.

(b) For which values of k is $\lim_{x \to 2} f(x) = -\infty$?

(c) If $k = -2$, does $\lim_{x \to 2} f(x)$ equal ∞, $-\infty$, or neither? Explain.

1.14 / Answer key

Infinite Limits and Vertical Asymptotes

1 (D)

2 (A)

3 (a) $(-3, 3)$; (b) $-\infty$ (not a real-number limit); (c) $x = 3$ and $x = -3$: $\lim_{x \to 3^-} f(x) = -\infty$ and $\lim_{x \to -3^+} f(x) = -\infty$

4 (a) 270 thousand dollars; (b) ∞ (not a real-number limit); (c) As the percent removed approaches 100, the cost increases without bound: removing all of the pollutant is not possible at any finite cost

5 (a) ∞ (not a real-number limit); (b) $k < -2$; (c) Neither: for $x \neq 2$, $f(x) = \dfrac{1}{x-2}$, which goes to $-\infty$ from the left and ∞ from the right