1.14 / Infinite Limits and Vertical Asymptotes

Watch the lesson ➤

Connecting infinite limits and vertical asymptotes

Learning goal

Describe unbounded behavior near $x = a$ with one-sided limits found from the signs of the top and the bottom, and use these infinite limits to find and justify vertical asymptotes.

Key ideas

01 The sign of the bottom picks the direction: $f(x) = \dfrac{x-1}{(x-1)(x-3)}$. At 1, $(x-1)$ cancels and $\dfrac{1}{x-3} \to -\dfrac{1}{2}$: a real limit and no $f(1)$, removable, not an asymptote. At 3 the top heads to 2 and the bottom to 0: unbounded. $x \to 3^-$: $x - 3 < 0$, the bottom $\to 0^-$; think $\dfrac{2}{0^-} = -\infty$, so $\lim_{x\to 3^-} f(x) = -\infty$. $x \to 3^+$: the bottom $\to 0^+$, $\lim_{x\to 3^+} f(x) = \infty$. Down on the left, up on the right.

02 AP: "As $x \to 3^-$, the top $\to 2$ and the bottom $\to 0$ through negative values, so $\lim_{x\to 3^-} f(x) = -\infty$." Never write $\dfrac{2}{0}$. The direction comes from the **signs** of the top and the bottom.

03 A side heads to $\pm\infty$ as $x \to a$: $x = a$ is a vertical **asymptote**. Even power: in $\dfrac{x+5}{(x-1)^2}$, $(x-1)^2 > 0$ on both sides; think $\dfrac{6}{0^+}$, so $\lim_{x\to 1} = \infty$, one direction. Repeated: $\dfrac{x}{x^3} = \dfrac{1}{x^2}$, a factor stays. Cancel first: what stays decides.

04 No fraction, still an asymptote: $\ln(x-2)$ has domain $x > 2$; its graph is $\ln x$ moved right 2, and $\lim_{x\to 2^+} \ln(x-2) = -\infty$, so $x = 2$ is the asymptote, not $x = 0$. $\tan x = \dfrac{\sin x}{\cos x}$ blows up where $\cos x = 0$: near $\dfrac{\pi}{2}$, $\cos x \to 0^+$ on the left and 0^- on the right. An asymptote at every $x = \dfrac{\pi}{2} + k\pi$.

Remember

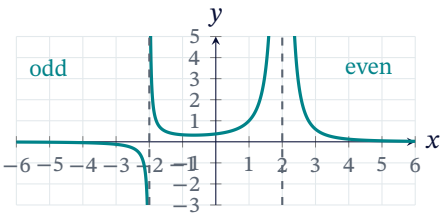
infinite limit
A one-sided or two-sided limit written as ∞ or $-\infty$. It is not a real number.

unbounded
The values of $f(x)$ get larger (or smaller) than any number as x gets close to a.

vertical asymptote
A vertical line $x = a$ where at least one one-sided limit of f is ∞ or $-\infty$.

even power
A power such as $(x-a)^2$ or $(x-a)^4$. It is never negative.

An odd power changes sign; an even power does not



Near $x = -2$ the factor $x + 2$ changes sign: one side is $-\infty$, the other ∞, and the limit does not exist. Near $x = 2$ the factor $(x-2)^2$ stays positive: both sides are ∞.

Class notes

1.14 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Describe unbounded behavior with limit notation and words: "$\lim_{x\to 3^-} f(x) = -\infty$: $f(x)$ decreases without bound as x approaches 3 from the left."	Do not do arithmetic with ∞: never write $\frac{5}{0} = \infty$ or $\infty - \infty = 0$.
Justify an asymptote with a one-sided limit, then conclude: "$\lim_{x\to 2^+} f(x) = \infty$, so $x = 2$ is a vertical asymptote."	Do not write a limit equal to $\frac{0}{0}$. Write "direct substitution gives the 0/0 form," then factor.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The top has a sign too

Code L12

✗ $\lim_{x\to 5^-} \dfrac{x-9}{x-5}$: "The bottom is negative, so the limit is $-\infty$."

✓ The top approaches -4, also negative. Negative over negative is positive: the limit is ∞.

02 Cancel one factor, then look at what is left

Code L9

✗ $\dfrac{x^2-6x}{(x-6)^2}$: "$x-6$ cancels, so $x=6$ is a hole."

✓ One $x-6$ is still in the bottom: $\dfrac{x}{x-6}$, and the top approaches 6. So $x=6$ is a vertical asymptote.

03 Infinity is not a number

Code L8

✗ "$\lim_{x\to -4} \dfrac{2}{(x+4)^2} = \infty$, so the limit exists."

✓ The equation says that the values increase without bound. The limit does not exist as a real number.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released FRQ from 2023–2026 asks for an infinite limit or a vertical asymptote. The 2012 BC Practice Exam multiple-choice question 11 lists "x = 2 is a vertical asymptote" as a false statement about a function with no asymptote. The items follow the CED: LIM-2.D.1 (infinite limits) and LIM-2.D.2 (asymptotic and unbounded behavior described with limits).

1.14 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
One-sided infinite limit: signs of the top and the bottom	Substitute first. If the top approaches a nonzero number and the bottom approaches 0, find the sign of the top and of the bottom on that side. Same signs: ∞. Different signs: $-\infty$.	
Two-sided: ∞, $-\infty$, or does not exist	Find both one-sided limits. Both ∞: write $\lim_{x\to a} f(x) = \infty$ (both $-\infty$: write $-\infty$). One side ∞ and the other $-\infty$: the limit does not exist. In every case the limit is not a real number.	
Even powers and repeated factors	Factor and cancel common factors first. If $x - a$ is left in the bottom, look at its power: an even power is positive on both sides of a, so both sides have the same sign; an odd power changes sign at a.	E01 · Practice 5
Vertical asymptotes of a rational function	Factor the top and the bottom and cancel common factors. Each zero of the bottom that is left gives a vertical asymptote; a zero that cancels completely gives a hole. If $x - a$ appears once in the bottom, there is no asymptote at a exactly when the top is 0 at a.	
Which function has a vertical asymptote?	For each function, find the one-sided limits at $x = a$. The line $x = a$ is a vertical asymptote exactly when at least one one-sided limit is ∞ or $-\infty$. A real-number limit (a hole, or no break) is not an asymptote.	E02 · Practice 1
Asymptotes of ln and tan	$\ln u \to -\infty$ as $u \to 0^+$: solve (inside) = 0 and approach from the side where the inside is positive. $\tan u = \frac{\sin u}{\cos u}$ has asymptotes where $u = \frac{\pi}{2} + k\pi$: as u approaches one from the left, $\tan u \to \infty$; from the right, $\tan u \to -\infty$.	Practice 3
Infinite limits read from a graph, a table, or a context	Follow each side to $x = a$. Values that grow without bound: ∞; values that drop without bound: $-\infty$. Write each one-sided limit. In context, say what the output does as the input approaches a, with units.	Practice 2, 4

Example 01

Video · E01

Let $g(x) = \dfrac{3x - 1}{x^2 + 2x + 1}$.
Find $\lim_{x \to -1} g(x)$.
Explain how you know.

> Can a square be negative?

Example 02

Video · E02

Which function has a vertical asymptote at $x = 3$?

A. $\dfrac{x - 3}{x^2 - 9}$
B. $\dfrac{x^2 - 9}{x - 3}$
C. $\ln(x - 3)$
D. $\dfrac{x - 3}{x^2 + 9}$

> Common factors first. Then look for a side that heads to $\pm\infty$.

VIDEO COMPLETE Check your example answers at the end of this booklet.

1.14 / Example answers

Infinite Limits and Vertical Asymptotes

E01 $-\infty$: the numerator approaches -4, and the denominator $(x+1)^2$ approaches 0 through positive values on both sides of -1, so $g(x)$ decreases without bound on both sides (think $\frac{-4}{0^+}$)

E02 (C)