1.13 / Practice workbook

Removing Discontinuities

Practice

Problems 1–6 / Show your reasoning

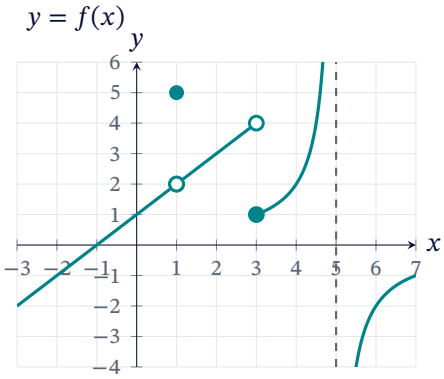
1 For $x \neq -2$, the function f satisfies $(x+2)f(x) = 2x^2 + x - 6$. If f is continuous at $x = -2$, what is $f(-2)$?

A. -7
B. 0
C. 1
D. It cannot be determined.

1.13 / AP-style free response

Problem 2 ➤ */ Show the work and the reasons a grader looks for.*

The graph of f is shown. The function is not continuous at $x = 1$, $x = 3$, and $x = 5$.



(a) At which of these values of x can f be redefined to make f continuous there?

(b) What value of f at that x makes f continuous?

(c) Explain why no value works at the other two.

1.13 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} ax + b, & x < 2 \\ 5, & x = 2 \\ ax^2 - 3b, & x > 2 \end{cases}$, where a and b are constants.

(a) Write two equations in a and b for continuity at $x = 2$.

(b) Find a and b. Justify your answer.

4 For which of the following can a value of $f(2)$ be defined so that f is continuous at $x = 2$?

I. $f(x) = \dfrac{x^2 - 2x}{|x - 2|}$

II. $f(x) = \dfrac{x^2 + 2x}{x^2 - 4}$

III. $f(x) = \dfrac{\sqrt{x + 7} - 3}{x - 2}$

A. I only
B. III only
C. I and III only
D. II and III only

1.13 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = \begin{cases} kx + 3, & x < 1 \\ 4, & x = 1 \\ x^2 + k, & x > 1 \end{cases}$, where k is a constant. A student says, "$k = 1$ makes f continuous at $x = 1$."

(a) Find $\lim_{x \to 1^-} f(x)$ in terms of k.

(b) Find $\lim_{x \to 1^+} f(x)$ in terms of k.

(c) Is the student correct? Is there any value of k that makes f continuous at $x = 1$? Explain.

6

Let $f(x) = \dfrac{\sin 6x}{2x}$ for $x \neq 0$. What value of $f(0)$ makes f continuous at $x = 0$?

A. $\dfrac{1}{3}$

B. 1

C. 3

D. 6

1.13 / Answer key

Removing Discontinuities

1 (A)

2 (a) At $x = 1$; (b) $f(1) = 2$, the limit; (c) At $x = 3$: $\lim_{x \to 3^-} f(x) = 4$ and $\lim_{x \to 3^+} f(x) = 1$ differ (jump). At $x = 5$: $\lim_{x \to 5^-} f(x) = \infty$ (infinite). No limit, so no value of f works.

3 (a) $2a + b = 5$ and $4a - 3b = 5$; (b) $a = 2$, $b = 1$: both one-sided limits and $f(2)$ are 5

4 (B)

5 (a) $k + 3$; (b) $k + 1$; (c) No. With $k = 1$ the left side is $4 = f(1)$, but the right side is 2. For every k, $k + 3 \neq k + 1$, so the one-sided limits always differ: no k works.

6 (C)